

Dissertation Abstract

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In early 1970s, Hugh Montgomery initialized the study of the pair correlation function $F(X, T)$ on zeros of Riemann zeta function and made the famous Strong Pair Correlation Conjecture on its asymptotic size when $T \leq X$. Later, assuming Riemann Hypothesis, Goldston and Montgomery proved that this and two other asymptotic formulas for second moments of primes in short intervals are equivalent. Recently, further assuming Hardy-Littlewood conjecture, Montgomery and Soundararajan derived a more precise asymptotic formula for one of the above second moments and worked on higher moments to get the conclusion that primes in short intervals obey normal distribution.

The first goal of this thesis is to make use of the more precise asymptotic formula described above to get more precise formula for the other second moment and the Strong Pair Correlation Conjecture. Our method is essentially that of Goldston & Montgomery but with explicit error terms. The second goal is to improve the asymptotic formulas for higher even moments which tells us how it deviates from normal distribution. Our third goal is then to get “equivalence” results on higher moments of primes in short intervals. For even moments, we use the same method as that for second moments. However, difficulties arise for odd moments. The best one can do is to assume good information on even moments and use Ghosh’s method. Finally, we try to get second order terms for $F(X, T)$ in both ranges $1 \leq X \leq T$ and $T \leq X$ in the hope to better understand how $F(X, T)$ changes in the transition when $X \approx T$. When $1 \leq X \leq T^{1-\epsilon}$, we make more precise the original calculation by Montgomery. When $T^{1-\epsilon} \leq X$, we use Goldston & Gonek’s method for mean values of Dirichlet polynomials and tails of Dirichlet series assuming Hardy-Littlewood Conjecture. From this, one can derive a more precise form of the Weak Pair Correlation Conjecture.