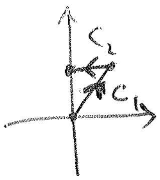
Math 223 Quiz #9 Name: Solution.

1. Evaluate $\int_C (x+y)\,dx - 2x\,dy$ where C consists of the line segments from $(0,0)$ to $(1,1)$ and from $(1,1)$ to $(0,1)$.



$C_1: \begin{cases} x=t \\ y=t \end{cases} \quad 0\le t\le 1 \qquad \int_0^1 (t+t)\cdot 1 - 2t\cdot 1\,dt = 0$

$C_2: \begin{cases} x=1-t \\ y=1 \end{cases} \quad 0\le t\le 1 \qquad \int_0^1 (1-t+1)(-1) - 2(1-t)\cdot 0\,dt$

$= -\int_0^1 2-t\,dt = \int_0^1 t-2\,dt$

$= \frac{t^2}{2} - 2t\Big|_0^1 = -\frac{3}{2}$

$\therefore \int_C = 0 + \frac{-3}{2} = \boxed{\frac{-3}{2}}.$

2. Let $\vec{F}(x,y) = \langle \sin y, x\cos y\rangle$ be a force field.
(a) Show that $\vec{F}$ has the independence of path property by finding the corresponding potential function $f(x,y)$.

$\frac{\partial f}{\partial x} = \sin y \longrightarrow f = x\sin y + g(y)$

$\frac{\partial f}{\partial y} = x\cos y \longrightarrow f = x\sin y + h(x)$

$\therefore \boxed{f = x\sin y + c}$

So, $\vec{F}$ has the independence of path property.

(b) Find the work required to move a particle from $(0,0)$ to $(1,1)$ via the curve $C: \vec{r}(t) = \langle te^{t-1}, t^2\rangle$, $0\le t\le 1$ under the force $\vec{F}$.

Work $= \int_C \vec{F}\cdot\vec{T}\,ds = \int_C P\,dx + Q\,dy$

$= f(1,1) - f(0,0)$ by Fundamental Theorem of Line Integral.

$= 1\cdot\sin 1 - 0\cdot\sin 0$

$= \boxed{\sin 1}.$

It is a good exercise to write out the line integral:

$$\int_0^1 \underbrace{\sin(t^2)}_{P}\underbrace{(e^{t-1}+te^{t-1})}_{dx} + \underbrace{te^{t-1}}_{Q}\underbrace{\cos(t^2)\cdot 2t}_{dy}\,dt$$