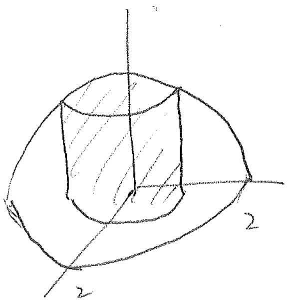
Math 223 **Quiz #8** **Name:** Solution

Consider the solid bounded by $x^2 + y^2 + z^2 = 4$, $x^2 + y^2 = 1$ and above the xy plane.

(a) Write down a triple integral for its volume in cylindrical coordinates.

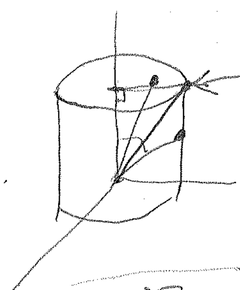
$r^2+z^2=4$ $\qquad$ $r^2=1$ or $r=1$



$$\boxed{\int_0^{2\pi}\int_0^1\int_0^{\sqrt{4-r^2}} 1 \cdot r \, dz \, dr \, d\theta}$$

(b) Write down a triple integral for its volume in spherical coordinates. (Hint: there should be two parts and $r = \rho \sin\phi$.)

Sometimes, the radius ends on the sphere but sometimes on the cylinder.



Intersection: $\quad r^2+z^2=4$ and $r=1$

$1+z^2=4 \Rightarrow z^2=3 \Rightarrow z=\sqrt{3}$

$\cos\phi = \frac{\sqrt{3}}{2} \Rightarrow \boxed{\phi = \frac{\pi}{6}}$

$r=1 \Rightarrow \rho\sin\phi = 1 \Rightarrow \rho = \frac{1}{\sin\phi}$

ends on sphere $\qquad$ ends on cylinder

$$\boxed{\int_0^{2\pi}\int_0^{\frac{\pi}{6}}\int_0^{2} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta + \int_0^{2\pi}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\int_0^{\frac{1}{\sin\phi}} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta}$$

(c) Write down a double integral for the surface area of the top surface of the solid.

The surface is part of the sphere.

$$z = \sqrt{4-x^2-y^2} = f(x,y)$$

$$\frac{\partial f}{\partial x} = \frac{-2x}{2\sqrt{4-x^2-y^2}}, \qquad \frac{\partial f}{\partial y} = \frac{-2y}{2\sqrt{4-x^2-y^2}}$$

$$\boxed{\therefore \text{Surface area} = \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1 + \frac{x^2}{4-x^2-y^2} + \frac{y^2}{4-x^2-y^2}} \, dy \, dx}$$