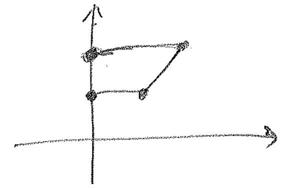
Math 223 **Quiz #6** **Name:** Solution

Let $f(x, y) = \sin(y^2)$ and R be the trapezoidal region with vertices $(0, 1)$, $(1, 1)$, $(2, 2)$ and $(0, 2)$. Consider the double integral

$$\iint_R f(x, y)\, dA.$$



(a) Write out the double integral where $dA = dx\, dy$.

$$\int_{y=1}^{2} \int_{x=0}^{x=y} \sin(y^2)\, dx\, dy$$

(b) Write out the double integral where $dA = dy\, dx$.

$$\int_0^1 \int_1^2 \sin(y^2)\, dy\, dx + \int_1^2 \int_{y=x}^{2} \sin(y^2)\, dy\, dx$$

(c) Write out the double integral in polar coordinates.

$y = 1 \Rightarrow r\sin\theta = 1 \Rightarrow r = \frac{1}{\sin\theta}$

$y = 2 \Rightarrow r\sin\theta = 2 \Rightarrow r = \frac{2}{\sin\theta}$

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{\frac{1}{\sin\theta}}^{\frac{2}{\sin\theta}} \sin(r^2\sin^2\theta)\, r\, dr\, d\theta$$

(d) Use (a), (b) or (c) to evaluate the double integral. (Hint: only one of them is easy to integrate.)

Use (a), $\displaystyle\int_1^2 \int_0^y \sin(y^2)\, dx\, dy$

$$= \int_1^2 \sin(y^2)\, x\Big|_0^y\, dy$$

$$= \int_1^2 y\sin(y^2)\, dy = \int_1^2 \sin u\, \frac{du}{2}$$

$$= \frac{-\cos u}{2}\Big|_1^2 = \frac{-\cos y^2}{2}$$

$$= \frac{\cos 1}{2} - \frac{\cos 4}{2}$$

$u = y^2$

$du = 2y\, dy$