Math 223 **Quiz #5** **Name:** Solution

Given $\frac{\partial f}{\partial x} = 3x^2 + 6xy$ and $\frac{\partial f}{\partial y} = 3x^2 + 2y$.

(a) Find f_{xx} and f_{yx}.

$f_{xx} = 6x + 6y$ //

$f_{yx} = 6x$ //

(b) Find $f(x,y)$ if $f(1,1) = 6$.

$\int \frac{\partial f}{\partial x}\,dx = x^3 + 3x^2y + g(y)$

$\int \frac{\partial f}{\partial y}\,dy = 3x^2y + y^2 + h(x)$

So, $f(x,y) = x^3 + 3x^2y + y^2 + C$

$6 = 1^3 + 3\cdot 1^2\cdot 1 + 1^2 + C$

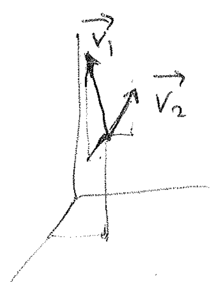
$C = 1$

$\boxed{\therefore\ f = x^3 + 3x^2y + y^2 + 1}$

(c) Find the normal vector of the tangent plane to the surface $z = f(x,y)$ at $(1,1,6)$.

$\left.\frac{\partial f}{\partial x}\right|_{(1,1)} = 3\cdot 1^2 + 6\cdot 1\cdot 1 = 9$

$\left.\frac{\partial f}{\partial y}\right|_{(1,1)} = 3\cdot 1^2 + 2\cdot 1 = 5$



$\vec{n} = \vec{V_1} \times \vec{V_2} = \langle 1, 0, 9\rangle \times \langle 0, 1, 5\rangle$

$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 9 \\ 0 & 1 & 5 \end{vmatrix} = \underline{\underline{\langle -9, -5, 1\rangle}}$

(d) Find all possible (x,y) where f has a local maximum or minimum.

$\begin{cases} \frac{\partial f}{\partial x} = 3x^2 + 6xy = 0 \\ \frac{\partial f}{\partial y} = 3x^2 + 2y = 0 \end{cases}$

Subtract,

$6xy - 2y = 0$

$3xy - y = 0$

$y(3x-1) = 0$

$y = 0$ or $3x = 1$

$y = 0$ or $x = \frac{1}{3}$.

When $y = 0$,

$3x^2 + 2\cdot 0 = 0 \Rightarrow x = 0$.

When $x = \frac{1}{3}$

$3\cdot\left(\frac{1}{3}\right)^2 + 2y = 0 \Rightarrow y = -\frac{1}{6}$.

So, the possible points are

$\underline{\underline{(0,0)}}$ and $\underline{\underline{\left(\frac{1}{3}, -\frac{1}{6}\right)}}$.