Math 223 Quiz #4 **Name:** Solution

1. $r^2 - z^2 = 1$ is an equation in cylindrical coordinates.

(a) Convert it both to rectangular coordinates and to spherical coordinates.

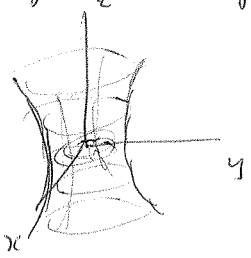
$\boxed{x^2+y^2-z^2=1}$

$(\rho\sin\phi\cos\theta)^2+(\rho\sin\phi\sin\theta)^2-(\rho\cos\phi)^2=1 \longrightarrow \boxed{(\rho\sin\phi)^2-(\rho\cos\phi)^2=1}$

(b) What kind of surface is it? Show your work and circle one answer.

A. Ellipsoid
B. Circular Paraboloid
C. Hyperboloid of one sheet
D. Hyperboloid of two sheets
E. Hyperbolic Paraboloid

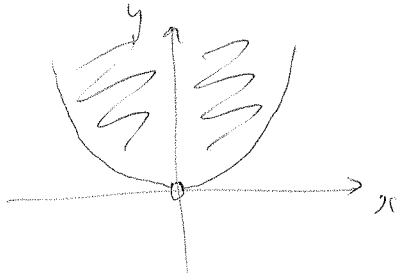
$z=0$: $x^2+y^2=1$, Circles
$x=0$: $y^2-z^2=1$, hyperbolas
$y=0$: $x^2-z^2=1$, hyperbolas.



2. Let $z = f(x,y) = \dfrac{\sqrt{y-x^2}}{x}$.

(a) What is the largest possible domain of f?

$y-x^2 \geq 0$ and $x \neq 0$.



(b) Sketch the level curves, $z = 1$ and $z = -1$, of the function f. Label your curves.

$z=1=\dfrac{\sqrt{y-x^2}}{x}$

$x=\sqrt{y-x^2}$

$x^2=y-x^2$

$y=2x^2$

$z=-1=\dfrac{\sqrt{y-x^2}}{x}$

$-x=\sqrt{y-x^2}$ same. So, one must be careful.

$x^2=y-x^2$

$y=2x^2$

In fact,