Math 223 **Quiz #3** **Name:** Solution.

Suppose that the **velocity** of a particle is given by $\vec{v}(t) = \vec{r}'(t) = \langle \cos t, 1, \sin t \rangle$.

(a) Find the unit normal vector $\vec{N}(t)$.

$$\vec{T} = \frac{\vec{v}}{v} = \frac{\langle \cos t, 1, \sin t \rangle}{\sqrt{\cos^2 t + 1^2 + \sin^2 t}} = \left\langle \frac{\cos t}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{\sin t}{\sqrt{2}} \right\rangle$$

$$\frac{d\vec{T}}{dt} = \left\langle \frac{-\sin t}{\sqrt{2}}, 0, \frac{\cos t}{\sqrt{2}} \right\rangle$$

$$\vec{N} = \frac{\frac{d\vec{T}}{dt}}{\left|\frac{d\vec{T}}{dt}\right|} = \frac{\left\langle \frac{-\sin t}{\sqrt{2}}, 0, \frac{\cos t}{\sqrt{2}} \right\rangle}{\sqrt{\left(\frac{-\sin t}{\sqrt{2}}\right)^2 + 0^2 + \left(\frac{\cos t}{\sqrt{2}}\right)^2}} = \langle -\sin t, 0, \cos t \rangle$$ //.

(b) Find the distance it travelled from $t = -\pi$ to $t = \pi$.

$$\text{Distance} = \text{Arc length} = \int_{-\pi}^{\pi} v(t)\,dt$$

$$= \int_{-\pi}^{\pi} \sqrt{2}\,dt$$

$$= \sqrt{2}t \Big|_{-\pi}^{\pi} = \sqrt{2}\pi - -\sqrt{2}\pi = 2\sqrt{2}\pi$$ //.

(c) Find the position of the particle at $t = \pi$ if $\vec{r}(-\pi) = \langle 1, 2, 3 \rangle$.

$$\vec{r}(t) = \int \vec{v}(t)\,dt$$

$$= \langle \sin t + C_1, t + C_2, -\cos t + C_3 \rangle$$

$$\langle 1, 2, 3 \rangle = \langle \sin(-\pi) + C_1, -\pi + C_2, -\cos(-\pi) + C_3 \rangle$$

$$= \langle C_1, -\pi + C_2, 1 + C_3 \rangle$$

$$\therefore C_1 = 1, \quad C_2 = 2 + \pi, \quad C_3 = 2$$

$$\vec{r}(t) = \langle \sin t + 1, t + 2 + \pi, -\cos t + 2 \rangle$$

$$\therefore \vec{r}(\pi) = \langle \sin \pi + 1, \pi + 2 + \pi, -\cos \pi + 2 \rangle$$

$$= \langle 1, 2 + 2\pi, 3 \rangle$$ //.