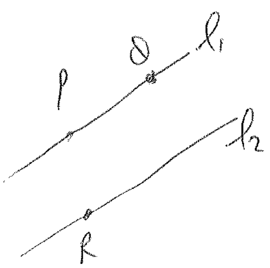
Math 223 **Quiz #2** **Name:** Solution

1. Given P $(1,2,3)$, Q $(3,-1,1)$ and R $(1,1,1)$. Let l_1 be the line through P and Q.

(a) Find the symmetric equation of the line, l_2, containing R and parallel to l_1.



$\overrightarrow{PQ} = \langle 3-1, -1-2, 1-3 \rangle = \langle 2, -3, -2 \rangle$

$$l_2: \begin{cases} x = 1 + 2t \\ y = 1 - 3t \\ z = 1 - 2t \end{cases}$$

$$\boxed{\frac{x-1}{2} = \frac{y-1}{-3} = \frac{z-1}{-2}}$$

(b) Find the equation of the plane containing the lines l_1 and l_2.

$\overrightarrow{PQ} = \langle 2, -3, -2 \rangle$

$\overrightarrow{PR} = \langle 0, -1, -2 \rangle$

$$\vec{n} = \overrightarrow{PQ} \times \overrightarrow{PR} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & -3 & -2 \\ 0 & -1 & -2 \end{vmatrix} = \vec{i}\begin{vmatrix} -3 & -2 \\ -1 & -2 \end{vmatrix} - \vec{j}\begin{vmatrix} 2 & -2 \\ 0 & -2 \end{vmatrix} + \vec{k}\begin{vmatrix} 2 & -3 \\ 0 & -1 \end{vmatrix}$$

$$= 4\vec{i} + 4\vec{j} - 2\vec{k}$$

So, $4x + 4y - 2z = d$

Since $(1,1,1)$ is on the plane

$4 \cdot 1 + 4 \cdot 1 - 2 \cdot 1 = d$

$d = 6$

$$\boxed{\begin{aligned} &\therefore\ 4x + 4y - 2z = 6 \\ &\text{or}\ \ 2x + 2y - z = 3. \end{aligned}}$$

2. $\vec{r}(t) = \langle t\sin t, t\cos t, t \rangle$. Show that the curve $\vec{r}(t)$ is on the surface $x^2 + y^2 = z^2$.

$x^2 + y^2 = (t\sin t)^2 + (t\cos t)^2$

$= t^2(\sin^2 t + \cos^2 t)$

$= t^2 \cdot 1$

$= z^2.$