Math 223 Quiz #1 **Name:** Solution

Given the points O $(0,0,0)$, A $(1,1,1)$ and B $(1,2,-3)$.

(a) Find $\overrightarrow{AB}$.

$$\overrightarrow{AB} = \langle 1-1, 2-1, -3-1 \rangle = \langle 0, 1, -4 \rangle$$

(b) Find $\text{comp}_{\overrightarrow{OB}}\overrightarrow{OA}$.

$$\text{comp}_{\overrightarrow{OB}}\overrightarrow{OA} = |\overrightarrow{OA}|\cos\theta = \frac{|\overrightarrow{OA}||\overrightarrow{OB}|\cos\theta}{|\overrightarrow{OB}|} = \frac{\overrightarrow{OA}\cdot\overrightarrow{OB}}{|\overrightarrow{OB}|}$$

$$= \frac{\langle 1,1,1\rangle \cdot \langle 1,2,-3\rangle}{\sqrt{1^2+2^2+(-3)^2}}$$

$$= \underline{\underline{0}}$$

i.e. $\overrightarrow{OA} \perp \overrightarrow{OB}$.

(c) Find the area of the triangle OAB.

$$\text{Area of } \triangle OAB = \frac{1}{2}|\overrightarrow{OA} \times \overrightarrow{OB}|$$

$$= \frac{1}{2}\left|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 1 & 2 & -3 \end{vmatrix}\right|$$

$$= \frac{1}{2}\left|\vec{i}\begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} - \vec{j}\begin{vmatrix} 1 & 1 \\ 1 & -3 \end{vmatrix} + \vec{k}\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}\right|$$

$$= \frac{1}{2}|-5\vec{i} + 4\vec{j} + \vec{k}|$$

$$= \frac{1}{2}\sqrt{(-5)^2+4^2+1^2}$$

$$= \underline{\underline{\frac{1}{2}\sqrt{42}}}$$