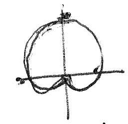


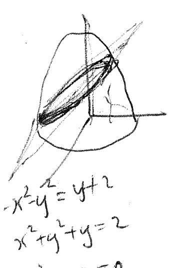
$\int_0^{2\pi}\int_0^{1+\sin\theta} 1\cdot r\,dr\,d\theta$

$Mass = \int_0^{2\pi}\int_0^{1+\sin\theta} r\cdot r\,dr\,d\theta$

- Polar Coordinates: Find the area of the region R bounded by $r = \sin\theta + 1$.
- Mass and Center of mass: Find the mass and center of mass of R above if $\delta(x, y) = \sqrt{x^2 + y^2}$.

$\bar{x} = 0$ by symmetry

$\bar{y} = \frac{1}{m}\int_0^{2\pi}\int_0^{1+\sin\theta} r\sin\theta \cdot r^2\,dr\,d\theta$.

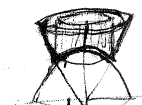


- Triple Integrals: Set up a triple integral for the mass of the solid bounded by $z = 4 - x^2 - y^2$ and $z = y + 2$ if $\delta(x, y, z) = z$. $\int\int\int_{y+2}^{4-x^2-y^2} z\,dz\,dx$

$-x^2 - y^2 = y + 2$

$x^2 + y^2 + y = 2$

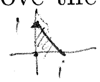
$y^2 + y - 2 = 0$



- Cylindrical and Spherical coordinates: Set up a triple integral (in cylindrical and spherical coordinates) for the volume of the solid: above the cone $z = r$, above the sphere $r^2 + z^2 = 1$ and below the plane $z = 4$.

$\int_0^{2\pi}\int_0^{\frac{1}{\sqrt{2}}}\int_{\sqrt{1-r^2}}^{4} r\,dz\,dr\,d\theta + \int_0^{2\pi}\int_{\frac{1}{\sqrt{2}}}^{4}\int^{z} r\,dr\,dz\,d\theta$

$\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_1^{\frac{4}{\cos\phi}} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$

- Parametric surfaces and surface area: Find the surface area of $\vec{r}(u, v) = \langle u + v, u - v, u^2 + v^2\rangle$ with $0 \le u \le 1$ and $0 \le v \le 1$. Find the area of the portion of the sphere $\rho = 1$ above the triangle with vertices $(0, 0)$, $(1, 0)$ and $(0, 1)$. $\int_0^1\int_0^1 \left|\frac{\partial\vec{r}}{\partial u}\times\frac{\partial\vec{r}}{\partial v}\right| du\,dv$  $\int_0^1\int_0^{1-x} \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\,dy\,dx$

Vector Calculus: Chapter 14

$\left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec{k} = \vec{0}$.

$1 + 1 = 2$

- Vector field: Draw the vector field $\vec{F}(x, y) = \langle x - y, x + y\rangle$. Find div$\vec{F}$ and curl$\vec{F}$. If $f(x, y, z) = x + y^2 + z^3$, find its gradient vector field. $\vec{\nabla} f = \langle 1, 2y, 3z^2\rangle$.

$\vec{\nabla}\times\langle x-y, x+y, 0\rangle$

- Line integrals: Find $\int_C x\,ds$, $\int_C x\,dx$, $\int_C x\,dy$, $\int_C x\,dz$, ... if C is the line joining $(1, 1, 1)$ and $(1, 2, 3)$. $\begin{cases} x = 1 \\ y = 1 + t \\ z = 1 + 2t \end{cases}$ $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\,dt$.
- Work $\int_C \vec{F}\cdot\vec{T}\,ds = \int_C P\,dx + Q\,dy + R\,dz$. Find the work done by the force field $\vec{F} = \langle y, z, x\rangle$ along the curve $x = t^3$, $y = t^2$, $z = t$, $0 \le t \le 2$.

$\int_C y\,dx + z\,dy + x\,dz$

$= \int_0^2 t^2\cdot 3t^2 + t\cdot 2t + t^3\cdot 1\,dt$

- Independence of path and fundamental theorem of line integral: Look at Quiz #9. Show that $\vec{F}(x, y, z) = \langle 2x - y - z, 2y - x, 2z - x\rangle$ has the independence of path property. Use it to find the work done by $\vec{F}$ from $(1, 1, 1)$ to $(1, 2, 3)$.

potential fcn $= x^2 + y^2 + z^2 - xy - xz + C$.

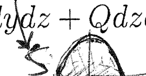
- Green's Theorem: $\vec{F}(x, y) = \langle y^2, 2x - 3y\rangle$. Suppose $\vec{F}$ is a force field. Find the work done along the circle $x^2 + y^2 = 9$ in positive orientation. Find the outward flux of $\vec{F}$ across the circle $x^2 + y^2 = 9$. Find the circulation of $\vec{F}$ around the circle $x^2 + y^2 = 9$.

Work = Circulation

$= \int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dy\,dx$

Flux $= \int\int \left(\frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y}\right) dy\,dx$.

- Surface integrals: $\vec{F} = \langle x, 0, z\rangle$. Find $\int\int_S \vec{F}\cdot\vec{n}\,dS$ where S is the upper hemisphere $z = \sqrt{1 - x^2 - y^2}$. (Hint: $\int\int_S \vec{F}\cdot\vec{n}\,dS = \int\int_S P\,dy\,dz + Q\,dz\,dx + R\,dx\,dy$)



$\int\int_S x\,dy\,dz + z\,dx\,dy$

in terms of y, z

$= \int_{-1}^{1}\int_0^{\sqrt{1-y^2}} \sqrt{1-y^2-z^2}\,dz\,dy + \int_{-1}^{1}\int_0^{\sqrt{1-y^2}} -\sqrt{1-y^2-z^2}\,dz\,dy$

$+ \int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \sqrt{1-x^2-y^2}\,dy\,dx$

- Divergence Theorem $\int\int_S \vec{F}\cdot\vec{n}\,dS = \int\int\int_R \text{div}\vec{F}\,dV$: $\vec{F} = \langle x, y, z\rangle$. Find the flux of $\vec{F}$ across the surface $S : x^2 + y^2 + \frac{z^2}{4} = 1$. $\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}\int_{-2\sqrt{1-x^2-y^2}}^{2\sqrt{1-x^2-y^2}} 3\,dz\,dy\,dx$

REMEMBER: FINAL EXAM, April 30 (Fri), 4-7 pm at Rock 301

Extra office hours: April 27 (Tue) 2-3:30 pm at Math Gala (Ballroom of Thwing), April 28 (Wed) 2-4 pm, April 29 (Thu) 2-4 pm.