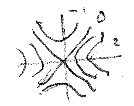
Math 223 Final exam review

Basics: Chapter 11

$\langle 3,6,12\rangle$ -11 $\langle 6,3,-3\rangle$

- Vectors: $\vec{a} = \langle 1,3,5\rangle$ and $\vec{b} = \langle -1,0,-2\rangle$. What is $2\vec{a}-\vec{b}$? $\vec{a}\cdot\vec{b}$? $\vec{b}\times\vec{a}$?
  $\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = |\vec{b}|\cos\theta = \frac{-11}{\sqrt{35}}$ = $\text{comp}_{\vec{a}}\vec{b}$? $\vec{a}_\perp$? $\vec{a}_\parallel$? $\vec{a}_\parallel = \frac{-11}{\sqrt{5}}\frac{\langle 1,3,5\rangle}{\sqrt{35}}$ $\vec{a}_\perp = \vec{a}-\vec{a}_\parallel$.
  Angle between $\vec{a}$ and $\vec{b}$? Area of parallelogram spanned by $\vec{a}$ and $\vec{b}$? Volume of parallelepiped and pyramid. ← Triple scalar product $\vec{a}\cdot(\vec{b}\times\vec{c})$.
- Lines and Planes: Find the parametric equations and symmetric equations of the line l_1, through $(1,2,3)$ and $(1,-1,1)$? Does l_1 intersect with l_2: $\frac{x-1}{1} = \frac{y-5}{2} = \frac{z-5}{3}$? Find the equation of the plane containing l_1 and l_2.
  $\begin{cases} x=1 \\ y=2-3t \\ z=3-2t \end{cases}$ $\boxed{\text{at } (1,5,5)}$ $-5x-2y+3z=0$
  $\vec{n} = \langle 0,-3,-2\rangle\times\langle 1,2,3\rangle = \langle -5,-2,3\rangle$
- Parametric curves: $\vec{r}(t) = \langle t,\cos t, e^t\rangle$. Find its velocity, speed, acceleration, $\vec{T}$, $\vec{N} = \frac{d\vec{T}/ds}{|d\vec{T}/ds|}$. Find its arclength from $-1\le t\le 2$. What is arclength parametrization? Find its curvature $\kappa = |\frac{d\vec{T}}{ds}| = \frac{|\vec{r}'(t)\times\vec{r}''(t)|}{|\vec{r}'(t)|^3}$. Find a_T and a_N.
  $\vec{r}' = \langle 1,-\sin t, e^t\rangle$, $v = \sqrt{1+\sin^2 t+e^{2t}}$ $\vec{a} = \langle 0,-\cos t, e^t\rangle$ $\vec{T} = \frac{\vec{v}(t)}{v(t)}$
- Surfaces: Identify the following surfaces: $z=1$, $z=x+y$, $z=x^2+y^2$, paraboloid
  elliptic cone → $z^2 = x^2+2y^2$, $x^2-y^2+z^2=1$, $x^2-y^2-z^2=1$, ...
  hyperboloid of one sheet hyperboloid of two sheets
- Cylindrical and Spherical coordinates: Convert $(x,y,z) = (-1,-2,-3)$ into (r,θ,z) and (ρ,θ,ϕ). (Be careful about θ and ϕ).
  $r=\sqrt{5}$, $\theta = \tan^{-1}2$ but $\pi<\theta<\frac{3\pi}{2}$ $\rho=\sqrt{14}$, $\phi = \cos^{-1}\frac{-3}{\sqrt{14}}$.

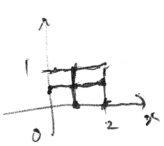
Derivatives: Chapter 12



hyperbolas.

- Level curves/surfaces: Draw some level curves for $f(x,y)=x^2-y^2$.
- Partial derivatives: $f(x,y,z) = xe^{xyz}$. Find f_x, f_y, f_z, f_{xz}, ...
- Linear Approximation: $f(1,1)=1$, $\frac{\partial f}{\partial x}=-2$, $\frac{\partial f}{\partial y}=3$. $f(1.2,1)\approx$? $\boxed{0.6}$ $f(1.2,0.9)\approx$? $\boxed{0.3}$
- Differential equations: Show that $u=e^t\sin x$ is a solution to $u_{xx}+u_{tt}=0$.
- Max and Min: The sum of three numbers is 100. What is the maximum of their product? Box problems...
  No global max! Local max at $x=y=z=33\frac{1}{3}$
- Chain rule: $f(x,y)=\sin(xy)$, $x=p+q+r$, $y=pqr$. Find $\frac{\partial f}{\partial p}$, $\frac{\partial f}{\partial q}$, $\frac{\partial f}{\partial r}$. Find $\frac{\partial z}{\partial x}$ if z is given implicitly by $\sin(xyz)-x^2z=1$.
  $zy\cos(xyz)+xy\cos(xyz)\frac{\partial z}{\partial x} - 2xz - x^2\frac{\partial z}{\partial x}$
  $y\cos(xyz)\frac{\partial xyz}{\partial x} - 2xz - x^2\frac{\partial z}{\partial x}=0 \Rightarrow \frac{\partial z}{\partial x} = \frac{2xz-zy\cos(xyz)}{xy\cos(xyz)-x^2}$.
- Directional derivatives and Gradient: $D_{\vec{u}}f = \vec{\nabla}f\cdot\vec{u}$. Note that it is closely related to linear approximation. Let $f(x,y,z)=xy+e^{yz}$. What is $D_{\vec{u}}f(1,1,1)$ in the direction $\langle 1,2,3\rangle$? In which direction does f increase the fastest when $(x,y,z)=(1,1,1)$?
  $\nabla f = \langle y, x+ze^{yz}, ye^{yz}\rangle$ $\nabla f(1,1,1) = \langle 1,1+e,e\rangle$ $D_{\vec{u}}f = \langle 1,1+e,e\rangle\cdot\frac{\langle 1,2,3\rangle}{\sqrt{14}}$
  gradient. $\frac{\langle 1,1+e,e\rangle}{\sqrt{1^2+(1+e)^2+e^2}}$

Integrals: Chapter 13



- Double Riemann Sums: Estimate $\int\int_R \sin(x+y)dA$ where R is the rectangle $0\le x\le 2$, $0\le y\le 1$ with $\Delta x = 1$ and $\Delta y = 0.5$ using the lower right corner.
  $[\sin(1+0)+\sin(1+0.5)+\sin(2+0)+\sin(2+0.5)]\cdot 1\cdot 0.5$
- Double Integrals: Set up the double integral $\int\int_R \sin(x+y)dA$ where R is the region bounded by $y=\frac{1}{x}$, $y=\frac{2}{x}$, $x=1$ and $x=2$. Switch the order of integration.
  $\int_1^2\int_{\frac{1}{x}}^{\frac{2}{x}} \sin(x+y)\,dy\,dx \longrightarrow \int_1^2\int_1^{\frac{2}{y}} \sin(x+y)\,dx\,dy + \int_{\frac{1}{2}}^1\int_{\frac{1}{y}}^2 \sin(x+y)\,dx\,dy.$