Math 122 Quiz #8 **Name:** Solution 8:30 / 11:30

1. Suppose that $\vec{a} = \vec{i} + 2\vec{j}$, $\vec{b} = -2\vec{i} - 3\vec{j}$ and $\vec{c} = 7\vec{i} + 8\vec{j}$.

(a) Find $2\vec{a} - \vec{b}$. $= 2(\vec{i} + 2\vec{j}) - (-2\vec{i} - 3\vec{j})$

$= 2\vec{i} + 4\vec{j} + 2\vec{i} + 3\vec{j} = \underline{4\vec{i} + 7\vec{j}}$

(b) Find $|\vec{c}|$ and a vector of length 3 that has the opposite direction as $\vec{c}$.

$|\vec{c}| = \sqrt{7^2 + 8^2}$ Vector $= \frac{-3}{\sqrt{113}} \cdot (7\vec{i} + 8\vec{j})$ //

$= \sqrt{113}$ //

(c) Are $\vec{a}$ and $\vec{b}$ perpendicular? Show your work.

$\vec{a} \cdot \vec{b} = (1)(-2) + (2)(-3) = -8 \neq 0$

So, $\vec{a}$ and $\vec{b}$ are not perpendicular.

(d) Write $\vec{c}$ as a linear combination of $\vec{a}$ and $\vec{b}$. (Hint: $\vec{c} = x\vec{a} + y\vec{b}$)

$\langle 7, 8 \rangle = x\langle 1, 2 \rangle + y\langle -2, -3 \rangle$

$= \langle x - 2y, 2x - 3y \rangle$

$\begin{cases} x - 2y = 7 & \text{—— } ① \\ 2x - 3y = 8 & \text{—— } ② \end{cases}$

$2 \times ①$ $2x - 4y = 14$ —— ③

② − ③ $y = -6$

$x + 12 = 7 \Rightarrow x = -5$

So $\vec{c} = -5\vec{a} - 6\vec{b}$ //

2. Given two points $P(-1, 1, 2)$ and $Q(2, -1, 0)$.

(a) Find $\overrightarrow{PQ}$. $= \langle 2 - -1, -1 - 1, 0 - 2 \rangle = \langle 3, -2, -2 \rangle$ //

(b) Find the equation of a sphere centered at P and passing through Q.

Radius $= |\overrightarrow{PQ}| = \sqrt{3^2 + (-2)^2 + (-2)^2} = \sqrt{17}$

$(x+1)^2 + (y-1)^2 + (z-2)^2 = 17$ //