Math 122 **Quiz #~~6~~ 7** **Name:** Solution 8:30 / 11:30

1. (a) Find the 4-th degree Taylor polynomial, $P_4(x)$, of $f(x) = x^{3/2}$ at $x = 1$.

$f(x) = x^{3/2}$ $\quad f(1) = 1$

$f'(x) = \frac{3}{2}x^{1/2}$ $\quad f'(1) = \frac{3}{2}$

$f''(x) = \frac{3}{2}\cdot\frac{1}{2}x^{-\frac{1}{2}}$ $\quad f''(1) = \frac{3}{4}$

$f'''(x) = \frac{3}{4}\cdot -\frac{1}{2}x^{-3/2}$ $\quad f'''(1) = -\frac{3}{8}$

$f^{(4)}(x) = -\frac{3}{8}\cdot -\frac{3}{2}x^{-5/2}$ $\quad f^{(4)}(1) = +\frac{9}{16}$

$$P_4(x) = 1 + \frac{3}{2}(x-1) + \frac{3/4}{2!}(x-1)^2 + \frac{-3/8}{3!}(x-1)^3 + \frac{9/16}{4!}(x-1)^4 \quad //$$

(b) Use $P_4(x)$ above to approximate $0.9^{3/2}$.

$$P_4(0.9) = 1 + \frac{3}{2}(-0.1) + \frac{3}{8}(-0.1)^2 - \frac{1}{16}(-0.1)^3 + \frac{3}{128}(-0.1)^4 = \boxed{0.853814 8}$$

2. Suppose that the Taylor series of $\tan^{-1} x$ at $x = 0$ is $x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}\cdot x^{2n-1}$ ~~$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n+1} x^{2n+1}$~~.

(a) Find the Taylor series of $\tan^{-1} 2x^2$ at $x = 0$.

$$2x^2 - \frac{(2x^2)^3}{3} + \frac{(2x^2)^5}{5} - \frac{(2x^2)^7}{7} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1}(2x^2)^{2n-1} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}2^{2n-1}}{2n-1} x^{4n-2}$$

(b) When $x = 1$, $\frac{\pi}{4} = \tan^{-1} 1 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$. Does the series converge? Explain.

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{1}{2n-1} = 0$ ✓

$a_n \geq a_{n+1} \geq 0$?

$\frac{1}{2n-1} \geq \frac{1}{2(n+1)-1}$? $\Rightarrow \frac{1}{2n-1} \geq \frac{1}{2n+1}$

$2n+1 \geq 2n-1$ which is true. ✓

So, by alternating series test, the series converges.

(c) When $x = \sqrt{3}$, $\frac{\pi}{3} = \tan^{-1}\sqrt{3} = 1 - \frac{3^{3/2}}{3} + \frac{3^{5/2}}{5} - \frac{3^{7/2}}{7} + \ldots$. Does the series converge? Explain.

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{(\sqrt{3})^{2n-1}}{2n-1} = \lim_{n\to\infty} \frac{(\sqrt{3})^{2n-1}\cdot \ln 3 \cdot 2}{2} = \infty \neq 0$$

or you can say $(\sqrt{3})^{2n-1}$ is exponential which is bigger than $2n-1$, polynomial

By the n^{th} term test, the series diverges.