Math 122 Quiz #6 **Name:** Solution. 8:30 / 11:30

Do 4 out of 5 and show your works (Don't just write down the answers).

1. Consider $\{\frac{1+\sin n}{n}\}_{n=1}^{\infty}$. What is the limit of this sequence as $n \to \infty$?

$$\lim_{n\to\infty} \frac{1+\sin n}{n} = \lim_{n\to\infty} \frac{1}{n} + \lim_{n\to\infty} \frac{\sin n}{n}$$

$$-\frac{1}{n} \le \frac{\sin n}{n} \le \frac{1}{n}$$

$$= 0 + 0 \quad \text{because} \quad \underset{0}{\downarrow} \qquad \underset{0}{\downarrow} \quad \text{Squeeze Law!}$$

$$= 0$$

2. Consider $\{n \sin \frac{1}{n}\}_{n=1}^{\infty}$. What is the limit of this sequence as $n \to \infty$? (Hint: $n \sin \frac{1}{n} = \frac{\sin \frac{1}{n}}{\frac{1}{n}}$.

$$\lim_{n\to\infty} \frac{\sin\frac{1}{n}}{\frac{1}{n}} \left(\frac{0}{0}\right) = \lim_{n\to\infty} \frac{\cos\frac{1}{n}\cdot \cancel{\frac{-1}{n^2}}}{\cancel{\frac{-1}{n^2}}} = \lim_{n\to\infty} \cos\frac{1}{n} = \cos 0 = \underline{\underline{1}}$$

L'Hopital's rule

3. Convert 0.15555... into a fraction.

$$= \frac{1}{10} + \frac{5}{100} + \frac{5}{1000} + \cdots$$

$$= \frac{1}{10} + \frac{\frac{5}{100}}{1-\frac{1}{10}} = \frac{1}{10} + \frac{\frac{5}{100}}{\frac{9}{10}} = \frac{1}{10} + \frac{5}{90} = \frac{14}{90} = \underline{\underline{\frac{7}{45}}}.$$

4. Determine the convergence of $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$.

$$\int_2^\infty \frac{1}{x \ln x}\,dx = \int_{x=2}^{\infty} \frac{1}{u}\,du = \lim_{b\to\infty} \int_{x=2}^{b} \frac{1}{u}\,du$$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$$= \lim_{b\to\infty} \ln u \Big|_{x=2}^{b} = \lim_{b\to\infty} \ln|\ln x|\Big|_{x=2}^{b}$$

$$= \lim_{b\to\infty} \ln\ln b - \ln\ln 2 = \infty$$

diverges!

5. Determine the convergence of $\sum_{n=1}^{\infty} \frac{1}{ne^n}$.

① Compare with $\sum_{n=1}^{\infty} \frac{1}{e^n} = \frac{\frac{1}{e}}{1-\frac{1}{e}}$ converges, And $\sum_{n=1}^{\infty} \frac{1}{ne^n} \le \sum_{n=1}^{\infty} \frac{1}{e^n}$.

So $\sum_{n=1}^{\infty} \frac{1}{ne^n}$ converges, too.

② Compare with $\sum_{n=1}^{\infty} \frac{1}{n \cdot n}$ (as $e^n \ge n$)

To prove $e^x \ge x$ or $e^x - x \ge 0$ rigorously, you may want to use derivatives

$f(x) = e^x - x$

$f'(x) = e^x - 1 \ge 0$ when $x \ge 0$. So, $f(x)$ is an increasing function.

$f(x) \ge f(0) = 1 \ge 0$!