Math 122 **Quiz #5** **Name:** Solution. 8:30 / 11:30

1. Express the polar equation $r^2 \sin\theta = \cos\theta$ in rectangular form.

$r^2 \cdot r\sin\theta = r\cos\theta$

$\boxed{(x^2+y^2)\cdot y = x}$

2. A tank of volume 1000 L contains salt water of concentration 0.2 kg/L at the beginning. Pure water is pumped into the tank at the rate of 5 L/min, and the mixture - kept uniform by stirring - is pumped out at the same rate.

(a) What is the amount of salt (kg) at the beginning? $1000\ L \times 0.2\,kg/L = \underline{200\ kg}$.

(b) Let $S(t)$ be the amount of salt in the tank at time t. Write down a differential equation satisfied by S.

$\frac{dS}{dt} = \text{rate in} - \text{rate out} = 0 - 5\ L/min \times \frac{S}{1000}\ kg/L$

(c) How long will it be until only 10 kg of salt remains in the tank?

$\frac{dS}{dt} = -\frac{S}{200}$

$\int \frac{dS}{S} = \int -\frac{1}{200}\,dt$

$\ln|S| = -\frac{1}{200}t + C$

$S = e^{-\frac{1}{200}t + C}$

$S = A\cdot e^{-\frac{1}{200}t}$

$S(0) = 200$ from (a).

$200 = A\cdot e^{-\frac{1}{200}\cdot 0}$

$A = 200$.

So, $S = 200e^{-\frac{1}{200}t}$

Now, $10 = 200e^{-\frac{1}{200}t}$ $\rightarrow$ $\frac{1}{20} = e^{-\frac{1}{200}t}$

$\ln\frac{1}{20} = -\frac{1}{200}t$

$t = -200\ln\frac{1}{20}$

$\approx \underline{600\ \text{mins}}.$

3. Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 20y = 0$; $y(0) = 2$, $y'(0) = 0$.

$r^2 + 4r + 20 = 0$

$r = \frac{-4 \pm \sqrt{4^2 - 4\cdot 1\cdot 20}}{2\cdot 1}$

$= \frac{-4 \pm \sqrt{16-80}}{2}$

$= \frac{-4 \pm \sqrt{-64}}{2}$

$= \frac{-4 \pm 8i}{2} = -2 \pm 4i$

So, $y = e^{-2x}[C\cos 4x + D\sin 4x]$

$2 = e^{-2\cdot 0}[C\cos 4\cdot 0 + \cancel{D\sin 4\cdot 0}]$

$\boxed{C = 2}$

$y' = -2e^{-2x}[C\cos 4x + D\sin 4x] + e^{-2x}[-4C\sin 4x + 4D\cos 4x]$

$0 = -2e^{-0}[C\cdot\cos 0 + \cancel{D\sin 0}] + e^{-0}[\cancel{-4C\sin 0} + 4D\cos 0]$

$0 = -2C + 4D$

$0 = -4 + 4D \Rightarrow \boxed{D = 1}$

$$\boxed{y = e^{-2x}[2\cos 4x + \sin 4x]}$$