Math 122 Quiz #4 Name: Solution 8:30 / 11:30

1. Consider $\frac{dy}{dx} = 2x\sqrt{y-1}$, $y(0) = 2$.

(a) Use Euler's method with 2 steps (or step size of 1) to estimate $y(2)$.

$$f(x+h) \approx f(x) + f'(x)\cdot h \quad . \quad h=1$$

$$y(1) \approx y(0) + y'(0)\cdot 1 = 2 + 2\cdot 0\sqrt{2-1}\cdot 1 = 2$$

$$y(2) \approx y(1) + y'(1)\cdot 1 = 2 + 2\cdot 1\sqrt{2-1}\cdot 1 = \underline{4}.$$

(b) Solve the differential equation and find $y(2)$ exactly.

$$\frac{dy}{dx} = 2x\sqrt{y-1}$$

$$\int \frac{dy}{\sqrt{y-1}} = \int 2x\,dx$$

$$2\sqrt{y-1} = x^2 + C$$

$$\sqrt{y-1} = \frac{x^2}{2} + C$$

$$y-1 = \left(\frac{x^2}{2}+C\right)^2 \leftarrow \text{Be careful}$$

$$y = \left(\frac{x^2}{2}+C\right)^2 + 1$$

$$2 = \left(\frac{0^2}{2}+C\right)^2 + 1$$

$$2 = C^2 + 1$$

$$C = 1 \text{ or } \cancel{-1}$$

Note: from $\sqrt{y-1} = \frac{x^2}{2} + C$,

$\sqrt{2-1} = \frac{0^2}{2} + C$

So $C = 1$.

$$y = \left(\frac{x^2}{2}+1\right)^2 + 1$$

$$y(2) = \left(\frac{2^2}{2}+1\right)^2 + 1$$

$$= \underline{10}$$

2. Solve $x\frac{dy}{dx} = 2y + x^3\cos x$, $y(0) = 3$.

$$x\frac{dy}{dx} - 2y = x^3\cos x$$

$$\frac{dy}{dx} + \frac{-2}{x}y = x^2\cos x$$

$$e^{\int\frac{-2}{x}dx}\frac{dy}{dx} + e^{\int\frac{-2}{x}dx}\cdot\frac{-2}{x}y = e^{\int\frac{-2}{x}dx}\cdot x^2\cos x$$

$$\frac{d\left(e^{\int\frac{-2}{x}dx}\cdot y\right)}{dx} = e^{\int\frac{-2}{x}dx}\cdot x^2\cos x$$

$$\frac{d\left(e^{-2\ln x}\cdot y\right)}{dx} = e^{-2\ln x}\cdot x^2\cos x$$

$$e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2}$$

$$\frac{d(x^{-2}\cdot y)}{dx} = x^{-2}\cdot x^2\cos x = \cos x$$

$$x^{-2}\cdot y = \int \cos x\,dx$$

$$= \sin x + C$$

$$y = x^2(\sin x + C)$$

Be careful.

$$3 = 0^2\cdot(\sin 0 + C)$$

$$3 = 0$$

Impossible!!!

So, there is no solution.