

1. The velocity,  $v(t)$  feet per second, of a certain object satisfies  $\frac{dv}{dt} = 3v^{1/2}$ . Find  $v(t)$  if  $v(0) = 4$ ?

$$\frac{dv}{dt} = 3v^{1/2}$$

$$v = \left(\frac{3t}{2} + c\right)^2 \text{ be careful!}$$

$$\int \frac{dv}{v^{1/2}} = \int 3 dt$$

$$2v^{1/2} = 3t + c$$

$$v^{1/2} = \frac{3t}{2} + c$$

$$4 = \left(\frac{3 \cdot 0}{2} + c\right)^2$$

$$\text{So, } c = \pm 2$$

$$v = \left(\frac{3t}{2} \pm 2\right)^2$$

2. Carbon extracted from an ancient skull contained only one-third as much radioactive carbon 14 as carbon extracted from present-day bone. How old is the skull if the half-life of carbon 14 is 5700 years? (Recall:  $\frac{dN}{dt} = -kN$ )

$$\frac{dN}{dt} = -kN$$

Use half-life to find  $k$ .

$$\int \frac{dN}{N} = \int -k dt$$

$$\ln|N| = -kt + c$$

$$N = e^{-kt+c}$$

$$N = N_0 e^{-kt}$$

$$\frac{N_0}{2} = N_0 e^{-k(5700)}$$

$$\frac{1}{2} = e^{-k(5700)}$$

$$\ln \frac{1}{2} = -k \cdot 5700$$

$$k = 1.216 \times 10^{-4}$$

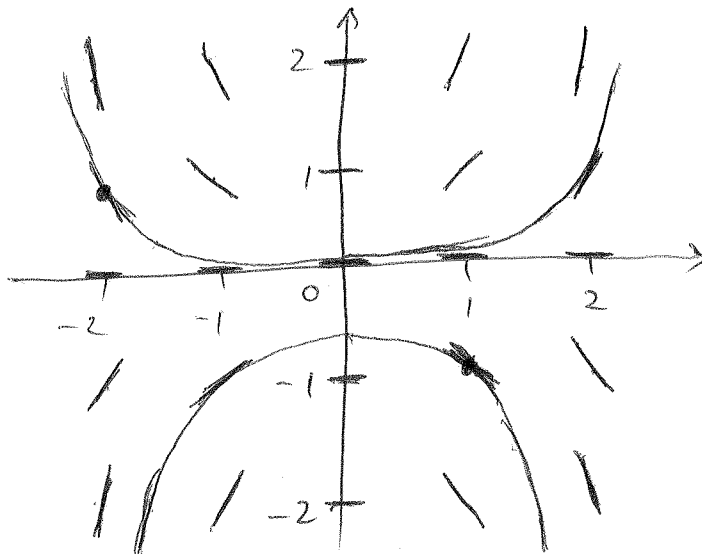
$$\frac{N_0}{3} = N_0 e^{-1.216 \times 10^{-4} t}$$

$$\frac{1}{3} = e^{-1.216 \times 10^{-4} t}$$

$$\ln \frac{1}{3} = -1.216 \times 10^{-4} t$$

$$t = 9034.29 \text{ years}$$

3. Draw the slope field of  $\frac{dy}{dx} = xy$ . Then sketch the solution curves corresponding to the initial conditions  $y(-2) = 1$  and  $y(1) = -1$ .



- Symmetric.
- Curves never cross one another.