Math 122 **Quiz #1** **Name:** Solution

1. Use $\int \sqrt{1-u^2}du = \frac{u}{2}\sqrt{1-u^2} + \frac{1}{2}\sin^{-1}u + C$ to find $\int \sqrt{4-x^2}dx$.

$$\int \sqrt{4-x^2}\,dx = \int \sqrt{4\left(1-\frac{x^2}{4}\right)}\,dx = 2\int \sqrt{1-\left(\frac{x}{2}\right)^2}\,dx$$

Let $u = \frac{x}{2}$, $du = \frac{1}{2}dx$ or $2du = dx$

Then $\int \sqrt{4-x^2}dx = 2\int \sqrt{1-u^2}\cdot 2du = 4\int \sqrt{1-u^2}du$

$= 4\left[\frac{u}{2}\sqrt{1-u^2} + \frac{1}{2}\sin^{-1}u + C\right]$

$= 2u\sqrt{1-u^2} + 2\sin^{-1}u + D = x\sqrt{1-\frac{x^2}{4}} + 2\sin^{-1}\frac{x}{2} + D$ //

2. Evaluate $\int e^x \sin x dx$. $\qquad \int fg' = f\cdot g - \int f'g$

$f = e^x \longrightarrow f' = e^x$

$g' = \sin x \longrightarrow g = -\cos x$

$\int e^x \sin x\,dx = e^x\cdot -\cos x - \int e^x\cdot -\cos x\,dx = -e^x\cos x + \int e^x \cos x\,dx$

$f = e^x \longrightarrow f' = e^x$

$g' = \cos x \longrightarrow g = \sin x$

$\int e^x \sin x\,dx = -e^x\cos x + e^x \sin x - \int e^x \sin x\,dx$

$2\int e^x \sin x\,dx = e^x(\sin x - \cos x) \Rightarrow \int e^x \sin x\,dx = \frac{e^x(\sin x - \cos x)}{2} + C$ //

3. Evaluate $\int \sin 2x \cos^2 2x\, dx$.

Let $u = \cos 2x$

$du = -\sin 2x \cdot 2\, dx$

$\int \sin 2x\cdot \cos^2 2x\cdot dx = -\frac{1}{2}\int u^2 du = -\frac{1}{6}u^3 + C$

$= -\frac{1}{6}\cos^3 2x + C$ //