

These problems are a selection of problems from old midterm exams that you can work in review for the second midterm (in class on Tuesday, March 4th). Solutions will be posted on the class web page: <http://filer.case.edu/pmg5/126/>. The blue-colored problems in the textbook's chapter review pages are also good review problems, and they cover some topics omitted here.

1 Compute the following improper integrals. Show all work necessary to justify your answer.

(a) $\int_{-\infty}^0 xe^{x^2} dx$

(b) $\int_1^{\infty} \frac{3x}{(x+1)^3} dx$

(c) $\int_0^1 \frac{1}{\sqrt{x}} dx$

2 Consider a random variable X whose probabilities are expressed using a density $f(x) = k\sqrt{x+10}$, $-6 \leq x \leq 6$.

- (a) For what value of k is the function $f(x)$ a valid probability density function?
- (b) Compute $\Pr(X \leq 0)$.
- (c) Find the expected value (or mean) $\mu = E(X)$.

3 Suppose an urn contains fourteen balls: five red and nine green. A ball is pulled from the urn, its color is noted, and then it is returned to the urn. This is repeated four times.

- (a) What is the probability that three of the four balls drawn from the urn are green?
- (b) Let X be the random variable that represents the number of green balls drawn. List the values of X together with the probability distribution.
- (c) Find $E(X)$, the expected value of this random variable X .

4 The starting salary of graduates from Standard American University is a normally distributed random variable with a mean of \$30,000 per year and a standard deviation of \$10,000 per year. (You may use a table of Z -values.)

- (a) What is the likelihood that a new graduate will start with a salary of more than \$50,000 per year.
- (b) Jessica has been given the choice of going to Standard U or enrolling in a training program at Jobs R Us. Clients of Jobs R Us hire graduates of the program for various positions. Each year 15% are hired as managers, 50% as clerks, 20% as assistants and 15% as technicians. Starting salaries are set at \$42,000 per year for managers, \$28,000 per year for clerks, \$25,000 per year for assistants and \$22,000 per year for technicians.
On the basis of starting salary alone, where should Jessica apply?

5 The amount of time required to serve a customer at a pizza restaurant during lunch time is an exponential random variable whose average is 5 minutes. The restaurant promises to give a full refund if the serving time for a customer is more than L minutes.

- (a) What is the value of the constant λ that is used in this exponential density function?
- (b) What is the probability that a customer is served in under 5 minutes?
- (c) What is L so that the restaurant only needs to refund 0.5% of the customers?

6 In a carnival game, a die is rolled that has three blue sides, two red sides and a green side. Fifty cents is placed on the color of your choice and the die is rolled. A match on blue pays fifty cents, a match on red pays \$1 and a match on green pays \$2. Let X be a random variable representing the payoff of a bet on red.

- (a) Make a probability table for X .
- (b) What is the average result? That is, what is the expected payoff?

- 7 Let $f(x) = \frac{1}{9}x^2$, $0 \leq x \leq 3$.
- Verify that $f(x)$ is a probability density function.
 - Find the expected value (or mean) $\mu = E(x)$.
 - Compute $\Pr(1 \leq X \leq 2)$.
- 8 The number of customers coming into a store was recorded every hour for 12 hours. The following observations were made.
- During 4 of the one-hour intervals, there was one customer.
 During 6 of the one-hour intervals, there were two customers.
 During 2 of the one-hour intervals, there were four customers.
- A one-hour interval is to be selected at random and the number of customers noted. Let X be the outcome. Then X is a random variable taking on values between 1 and 4.
- Write out a probability table for X .
 - Compute the expected value $E(X)$.
- 9 An appliance comes with a full 6-month warranty. It has been found that the time before the appliance experiences some sort of malfunction is an exponential random variable with mean 2 years.
- Find the exponential density function $f(x)$.
 - What percentage of appliances will malfunction during the warranty period?
- 10 A farmer has observed that the time to maturation of a certain crop is approximately normally distributed with mean, μ , of 60 days and standard deviation, σ , of 2 days. Use the attached table for standard normal distribution to find the percentage of plants that will mature in less than 57 days.
- 11 A random variable X is given by the following probability table:
- | | | | | |
|--------------|-----|-----|---|---|
| Outcome: | 0 | 1 | 2 | 3 |
| Probability: | 0.5 | 0.1 | | |
- Unfortunately, as you can see several of the entries have been lost. There is a note, however, that $E(X) = 1$. From this information complete the probability table.
- 12 Consider the random variable X with probability density function $f(x) = kx^2$, $0 \leq x \leq 2$.
- What is the value of k ?
 - Find the probability $\Pr(0 \leq X \leq 1)$.
 - What is the expected value $E(X)$?
 - What is the variance $\text{Var}(X)$?
- 13 The length of an adult of a certain species of worm is assumed to be a random variable X that is normally distributed with mean 250 mm and standard deviation 15 mm. Find the probability that an adult worm (of this species) is between 238 mm and 253 mm long. (You may use a table of Z -values.)
- 14 A charge on a certain laptop battery lasts, on average, for 100 minutes. Assume the battery life is an exponential random variable. What is the probability that the battery charge will last for two hours (120 minutes)?
- 15 Let $S = \{ 1, 2, 3, 4, x, y, z \}$ be a set of 7 numbers. We are told that the mean of this set is 2, the median is 2.5, and the mode is 4. Find x , y , and z .