These problems are a selection of problems from old midterm exams that you can work in review for the third midterm (in class on Tuesday, April 8th). Solutions will be posted by this weekend on the class web page `http://filer.case.edu/pmg5/126/`. The blue-colored problems in the textbook's chapter review pages are also good review problems, and they cover some topics omitted here.

1 Find the solution to the differential equation $y' = y^2 x e^{x^2}$ assuming $y(0) = 1$.

2 Consider the differential equation $y' = 2t(y - 2)$.

(a) Find all constant solutions (that is, solutions $y = C$) of the differential equation.

(b) For what value of k is the function $y = e^{t^2} + k$ a solution to this differential equation?

3 Let $y = Z(t)$ be the number of zebras at time t. Suppose that the rate of growth of the zebra population is proportional to the difference between 10,000 and the population at time t. When $Z(t)$ is small, the population is increasing.

(a) Write down a differential equation for $Z(t)$. State whether the proportionality constant is positive or negative.

(b) Suppose a new predator is introduced that kills 150 zebras per year. Modify the differential equation in (a) to account for this new information.

4 (a) Find the general solution to the following differential equation:

$$\frac{dy}{dx} = \frac{xy^2}{1 + 3x^2}.$$

(b) Find the general solution to the following differential equation:

$$xy' - 3y = x^3 \qquad \text{for } x > 0.$$

5 A boiled egg with initial temperature of 212° F is plunged into a water-filled container that is kept at a constant temperature of 40° F. The rate of change of the temperature of the egg, $f(t)$, at time t, in minutes, is proportional to the difference of the temperatures of the water and the egg.

(a) Set up an initial value problem that the is satisfied by the temperature of the cooling egg, as described above.

(b) Determine the sign of the constant of proportionality, with appropriate reasoning.

(c) What would be the temperature of the egg after one week?

6 (a) Solve the differential equation

$$y' = t^2 e^{-y} - e^{-y}$$

(b) Solve the initial value problem

$$\begin{aligned} y' &= 2y + e^{5t} \\ y(0) &= 1. \end{aligned}$$

7 A cup of coffee cools on a table. The coffee, when it is first placed on the table, is 170° F. After 5 minutes, the coffee has cooled to 160° F. The room temperature is a constant 70° F. Assume that the rate of change of the temperature of the coffee is proportional to the difference between the temperature of the room and the temperature of the coffee.

(a) Set up an initial value problem that is satisfied by the temperature of the cooling cup of coffee. **You do not need to solve this equation!**

(b) Determine the sign of the constant of proportionality in your equation from part (a). Explain your reasoning!

8 Suppose on this campus of 9952 students, one student returns from break with the flu virus and proceeds to share that virus with the student population. Assume that the virus spreads at a rate proportional to the product of the number of students presently infected and the number of students not yet infected. Determine the appropriate differential equation that models this situation. *Please do not try to solve this differential equation.*

9 (a) Write the system

$$\begin{aligned} x + y + z &= 3 \\ x - y + z &= 7 \\ x - y - z &= 1 \end{aligned}$$

in $AX = B$ format.

(b) Rewrite the system in the augmented matrix format $[\ A \mid B\ ]$.

(c) Use the method of Gaussian elimination to solve for x, y, and z, if they exist.

10 Suppose the graph of a quadratic equation $y = ax^2 + bx + c$ passes through the points $(1, 5)$, $(2, 13)$, and $(3, 25)$. Find a, b, c.

11 Find all the values of α and β so that the system

$$\begin{aligned} x + 3y - 2z &= 5 \\ 2x + 4y + 3z &= -3 \\ -x + y + \alpha z &= \beta \end{aligned}$$

(a) ...has a unique solution.

(b) ...has no solution.

(c) ...has infinitely many solutions.

12 Find the inverses of the following matrices, if these inverses exist:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 1 \\ 2 & 4 & 9 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 6 & 7 & 8 \\ 9 & 17 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 4 & 0 & -2 & 0 \\ 3 & 6 & 0 & -4 & 1 \\ 5 & 10 & 1 & -4 & 0 \\ 7 & 14 & 2 & -5 & 0 \\ 4 & 8 & 3 & -2 & 5 \end{bmatrix}$$

13 Which of the following 3×3 matrices are invertible? For those that are invertible, find their inverse and use it to solve the linear system $AX = B$, where

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}.$$

(a) $A = \begin{bmatrix} 1 & -4 & -6 \\ 0 & 1 & 4 \\ 0 & 0 & 2 \end{bmatrix}$ (b) $A = \begin{bmatrix} 1 & 5 & 6 \\ 0 & 1 & 2 \\ -1 & -3 & -2 \end{bmatrix}$ (c) $A = \begin{bmatrix} 0 & 3 & 8 \\ -1 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$

14 Solve the system

$$\begin{aligned} 2x_1 - x_2 + 5x_3 - 3x_4 &= 1 \\ 2x_1 - x_2 + x_3 - x_4 &= -1 \\ -3x_1 + 4x_2 - x_4 &= 0 \end{aligned}$$

by using row operations to put the augmented matrix $[\ A \mid B\ ]$ in reduced row echelon form.