EXAM THREE FORMULA SHEET

Differentiation:

$$\frac{d}{dx}(x^n) = nx^{n-1}(\text{if } n \neq 0)$$

$$\frac{d}{dx}(\ln(|x|)) = \frac{1}{x}$$

$$\frac{d}{dx}(e^{rx}) = re^{rx}$$

Product Rule:

$$(uv)' = u'v + uv'$$

Quotient Rule:

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

Chain Rule:

$$\frac{d}{dx}(f(g(x))) = f'(g(x))g'(x)$$

Integration:

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \text{ if } n \neq -1$$

$$\int \frac{1}{x}\,dx = \ln(|x|) + C$$

$$\int e^{rx}\,dx = \frac{1}{r}e^{rx} + C$$

Integration by Parts:

$$\int u\,dv = uv - \int v\,du$$

Substitution:

$$\int f(u(x))u'(x)\,dx = \int f(u)\,du$$

Matrices:
The inverse of a square matrix A is the matrix A^{-1} such that $A^{-1}A = AA^{-1} = I$:

$$\text{For a } 2 \times 2 \text{ matrix } A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, \qquad A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

In general, elimination (row reduction) may be used to make the following transformation: $[A \mid I\,] \to [\,I \mid A^{-1}]$. The three row operations are

- Interchange any two rows
- Multiply any row by a (nonzero) constant
- Add any multiple of one row to another

Separable: Turn into

$$f(y)\,dy = g(x)\,dx$$

then integrate both sides

First Order Linear:

$$y' + P(x)y = G(x) \qquad (SF)$$

has integrating factor

$$h(x) = e^{\int P(x)\,dx}.$$

Quadratic Formula: $ax^2 + bx + c = 0$ has roots

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Second Order Linear:

$$ay'' + by' + cy = 0 \qquad (H)$$

$$ar^2 + br + c = 0 \qquad (C)$$

- Case 1: $r_1 \neq r_2$, both real

$$y = Pe^{r_1 t} + Qe^{r_2 t}$$

- Case 2: $r_1 = r_2 = r$ is real

$$y = e^{rt}(P + Qt)$$

- Case 3: $r = \alpha \pm i\beta$

$$y = e^{\alpha t}(P\cos(\beta t) + Q\sin(\beta t))$$