Quiz Six Solutions

1 The given augmented matrices can be transformed using elimination (or elementary row operations) into the shown new, simpler ones. In each case, find the unknown constant (a, b, or c).

(a) $\left[\begin{array}{cc|c} 1 & a & 3 \\ 2 & 6 & 10 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & a & 3 \\ 0 & 1 & 4 \end{array}\right].$

Solution: Let's begin by trying to row reduce our original augmented matrix. We'll subtract 2 copies of row one from row two; this will make the 2 in the second row (below the pivot) zero:

$$\left[\begin{array}{cc|c} 1 & a & 3 \\ 2 & 6 & 10 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & a & 3 \\ 0 & 6-2a & 4 \end{array}\right] \qquad R_2 = r_2 - 2r_1.$$

Our second row now agrees with the given second row, provided $6 - 2a = 1$. This means that $a = \frac{5}{2} = 2.5$.

(b) $\left[\begin{array}{cc|c} 1 & 2 & b \\ -1 & 2 & 7 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & b \\ 0 & 1 & 2 \end{array}\right].$

Solution: As before, we begin by trying to row reduce our original augmented matrix. This time, we'll add row one to row two; this will make the -1 in the second row (below the pivot) zero:

$$\left[\begin{array}{cc|c} 1 & 2 & b \\ -1 & 2 & 7 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & b \\ 0 & 4 & b+7 \end{array}\right] \qquad R_2 = r_2 + r_1.$$

Our second row now does *not* agree with the given second row – we need to turn the 4 into a 1. We do this by continuing the row reduction, multiplying the second row by $\frac{1}{4}$:

$$\left[\begin{array}{cc|c} 1 & 2 & b \\ 0 & 4 & b+7 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & b \\ 0 & 1 & \frac{b+7}{4} \end{array}\right] \qquad R_2 = \tfrac{1}{4} \cdot r_2.$$

Now the second row agrees with the given second row provided $\frac{b+7}{4} = 2$, or $b = 1$.

(c) $\left[\begin{array}{cc|c} 1 & 2 & 3 \\ c & 1 & 2 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & 4 \end{array}\right].$

Solution: This is the trickiest: to perform our row reduction, we subtract c copies of row one from row two:

$$\left[\begin{array}{cc|c} 1 & 2 & 3 \\ c & 1 & 2 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1-2c & 2-3c \end{array}\right] \qquad R_2 = r_2 - c \cdot r_1.$$

To match the given second row, we must turn the $1-2c$ into a 1. We thus divide the second row by this term:

$$\longrightarrow \left[\begin{array}{cc|c} 1 & 2 & 3 \\ 0 & 1 & \frac{2-3c}{1-2c} \end{array}\right] \qquad R_2 = \frac{1}{1-2c}\cdot r_2.$$

This agrees with the given second row provided $\frac{2-3c}{1-2c} = 4$, or (after some algebra) $c = \frac{2}{5}$.

2 Find the solution of the linear system corresponding to the following augmented matrix. (If there is no solution, write "inconsistent.") You must use row reduction (elementary row operations) and you must show your work. Minimal credit will be given for simply writing down the answer.

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 12 \\ 4 & 0 & 2 & 12 \\ -2 & 1 & 2 & -9 \end{array}\right]$$

Solution: We begin our solution by using the given pivot in the first row and first column to zero out the remaining elements of the first column:

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 12 \\ 4 & 0 & 2 & 12 \\ -2 & 1 & 2 & -9 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 12 \\ 0 & -8 & 6 & -36 \\ 0 & 5 & 0 & 15 \end{array}\right] \qquad \begin{array}{l} R_2 = r_2 - 4r_1 \\ R_3 = r_3 + 2r_1. \end{array}$$

We then notice that the third row would make an ideal second row, as it has only a non-zero term (to the left of the vertical bar) in the second column. So we switch the second and third rows, then divide this new second row by 5 to make the new pivot equal to 1:

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 12 \\ 0 & 5 & 0 & 15 \\ 0 & -8 & 6 & -36 \end{array}\right] \qquad \begin{array}{l} R_2 = r_3 \\ R_3 = r_2 \end{array}$$

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 12 \\ 0 & 1 & 0 & 3 \\ 0 & -8 & 6 & -36 \end{array}\right] \qquad R_2 = \frac{1}{5}\cdot r_2.$$

We now make the -8 in the third row a zero by adding eight copies of the second row to the third:

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 12 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 6 & -12 \end{array}\right] \qquad R_3 = r_3 + 8\cdot r_2.$$

Finally we make the first non-zero term in the third row a 1 by dividing by the appropriate number:

$$\longrightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 12 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -2 \end{array}\right] \qquad R_3 = \tfrac{1}{6} \cdot r_3.$$

Now we backsolve: this corresponds to the linear system

$$\begin{aligned} x + 2y - z &= 12 \\ y \qquad &= 3 \\ z &= -2. \end{aligned}$$

Plugging $y = 3$ and $z = -2$ into the first equation gives us $x = 4$. That is, $(x, y, z) = (4, 3, -2)$.

3 Two groups of students go out to eat at Jalapeño Burrito Company. One group gets 4 burritos and 2 colas; their bill is \$29. The second group gets only 3 burritos, but 5 colas; their food costs \$27. How much does each burrito and each cola cost? (Assume that each burrito costs the same, and each cola costs the same.)

Solution: Let B be the price of a burrito and C be the price of a cola. Then we're told that $4B + 2C = \$29$ and $3B + 5C = \$27$. That is, we have the following system of equations

$$\begin{aligned} 4B + 2C &= 29 \\ 3B + 5C &= 27 \end{aligned}$$

or, equivalently, the following augmented matrix

$$\left[\begin{array}{cc|c} 4 & 2 & 29 \\ 3 & 5 & 27 \end{array}\right].$$

We'll perform the elimination by simply noting the actions on the right:

$$\left[\begin{array}{cc|c} 4 & 2 & 29 \\ 3 & 5 & 27 \end{array}\right] \longrightarrow \left[\begin{array}{cc|c} 1 & 0.50 & 7.25 \\ 3 & 5 & 27 \end{array}\right] \qquad R_1 = \tfrac{1}{4} \cdot r_1$$

$$\longrightarrow \left[\begin{array}{cc|c} 1 & 0.50 & 7.25 \\ 0 & 3.50 & 5.25 \end{array}\right] \qquad R_2 = r_2 - 3r_1$$

$$\longrightarrow \left[\begin{array}{cc|c} 1 & 0.50 & 7.25 \\ 0 & 1 & 1.50 \end{array}\right] \qquad R_2 = \tfrac{1}{3.50} \cdot r_2.$$

We can stop here and return to equations. The last augmented matrix represents the system of equations

$$\begin{aligned} B + 0.50C &= 7.25 \\ C &= 1.50 \end{aligned}$$

From this we find that $B + 0.50(1.50) = 7.25$, or $B = 6.50$. Thus burritos cost \$6.50 each and colas cost \$1.50 each.

Finally, let's check to make sure we've got the right answer. That is, let's plug in $B = \$6.50$ and $C = \$1.50$ into our original equations:

$$4B + 2C = 4(\$6.50) + 2(\$1.50) = \$26.00 + \$3.00 = \$29.00$$
$$3B + 5C = 3(\$6.50) + 5(\$1.50) = \$19.50 + \$7.50 = \$27.00$$

Thus we have the correct answer.