Quiz One Solutions

1 Evaluate the indefinite integral $\int x^3 \left(x^4 - 7\right)^{12} \, dx$.

Solution: Let us make the substitution $u = x^4 - 7$, so $\frac{du}{dx} = 4x^3$ and $du = 4x^3 \, dx$. We can then write $dx = \frac{du}{4x^3}$, so

$$\int x^3 \left(x^4 - 7\right)^{12} \, dx = \int x^3 u^{12} \, \frac{du}{4x^3} = \int \frac{1}{4} u^{12} \, du = \frac{1}{4} \int u^{12} \, du.$$

We can now integrate this then re-substitute to get an answer involving x:

$$\begin{aligned} \int x^3 \left(x^4 - 7\right)^{12} \, dx &= \frac{1}{4} \int u^{12} \, du = \frac{1}{4} \cdot \frac{1}{13} u^{13} + K \\ &= \frac{1}{4 \cdot 13} (x^4 - 7)^{13} + K \\ &= \frac{1}{52} (x^4 - 7)^{13} + K. \end{aligned}$$

This is our final answer.

2 Evaluate the indefinite integral $\int \frac{2x+1}{3x^2 + 3x - 4} \, dx$.

Solution: Here we make the substitution $u = 3x^2 + 3x - 4$, the denominator. (We're aiming for the integral $\int \frac{1}{u} \, du$, so we've selected u to be the given denominator.) Then $\frac{du}{dx} = 6x + 3$, so $du = (6x + 3) \, dx$ or $dx = \frac{1}{6x+3} \, du$. We then make our substitution to find that

$$\begin{aligned} \int \frac{2x+1}{3x^2 + 3x - 4} \, dx &= \int \frac{2x+1}{u} \cdot \frac{1}{6x+3} \, du = \int \frac{2x+1}{u} \cdot \frac{1}{3(2x+1)} \, du \\ &= \int \frac{1}{u} \cdot \frac{1}{3} \, du = \frac{1}{3} \int \frac{1}{u} \, du \end{aligned}$$

Now we integrate and change back from u to x:

$$\begin{aligned} \int \frac{2x+1}{3x^2 + 3x - 4} \, dx &= \frac{1}{3} \int \frac{1}{u} \, du = \frac{1}{3} \ln(|u|) + K \\ &= \frac{1}{3} \ln\left(|3x^2 + 3x - 4|\right) + K. \end{aligned}$$

This is our final answer.

3 In 2007, there were 340 students in a particular school district. Research indicates that the district expects this number will increase at a rate of $4t + 15$, where t is the number of years since 2007. Estimate the number of students in the district in 2011.

Solution: Let $N(t)$ be the number of students t years from the year 2007. Thus we're asked for $N(4)$, given that $N(0) = 340$ and $N'(t) = 4t + 15$. We simply integrate:

$$N(t) = \int (4t + 15)\ dt = 2t^2 + 15t + K.$$

In order to find the constant of integration K, we plug in $N(0) = 340$:

$$340 = N(0) = 2(0)^2 + 15(0) + K = K,$$

so $K = 340$ and $N(t) = 2t^2 + 15t + 340$. Thus the number of students in 2011 will be $N(4) = 2(4)^2 + 15(4) + 340 = 432$.

4 The depreciation rate of a new car follows the equation $V'(t) = -3000e^{-0.6t}$ where $V(t)$ is the value of the car and t years after purchase. If the car cost $\$24,000$ new, what is the value of the car after 4 years?

Solution: We're told that $V'(t) = -3000e^{-0.6t}$ and $V(0) = 24,000$ and asked for $V(4)$. This means we need to find $V(t)$, which we do by integrating:

$$V(t) = \int -3000e^{-0.6t}\ dt = -3000 \int e^{-0.6t}\ dt = -3000 \cdot \frac{1}{-0.6} e^{-0.6t} + K = 5000e^{-0.6t} + K.$$

To find the constant of integration K, we plug in $t = 0$ and $V(0) = 24,000$:

$$24,000 = V(0) = 5000e^{-0.6(0)} + K = 5000 + K,$$

so $K = 24,000 - 5000 = 19,000$. Thus $V(t) = 5,000e^{-0.6t} + 19,000$ and so $V(4) = 5,000e^{-0.6(4)} + 19,000$, or about $\$19,453.59$.