Quiz One Solutions

1 Evaluate the indefinite integral $\int x^2\left(x^3+5\right)^{15}\ dx$.

Solution: Let us make the substitution $u = x^3+5$, so $\frac{du}{dx} = 3x^2$ and $du = 3x^2\ dx$. We can then write $dx = \frac{du}{3x^2}$, so

$$\int x^2\left(x^3+5\right)^{15}\ dx = \int x^2u^{15}\ \frac{du}{3x^2} = \int \frac{1}{3}u^{15}\ du = \frac{1}{3}\int u^{15}\ du.$$

We can now integrate this then re-substitute to get an answer involving x:

$$\begin{aligned}\int x^2\left(x^3+5\right)^{15}\ dx = \frac{1}{3}\int u^{15}\ du &= \frac{1}{3}\cdot\frac{1}{16}u^{16}+K\\ &= \frac{1}{3\cdot 16}(x^3+5)^{16}+K\\ &= \frac{1}{48}(x^3+5)^{16}+K.\end{aligned}$$

This is our final answer.

2 Evaluate the indefinite integral $\int \frac{2x+1}{2x^2+2x+7}\ dx$.

Solution: Here we make the substitution $u = 2x^2+2x+7$, the denominator. (We're aiming for the integral $\int \frac{1}{u}\ du$, so we've selected u to be the given denominator.) Then $\frac{du}{dx} = 4x+2$, so $du = (4x+2)\ dx$ or $dx = \frac{1}{4x+2}\ du$. We then make our substitution to find that

$$\begin{aligned}\int \frac{2x+1}{2x^2+2x+7}\ dx &= \int \frac{2x+1}{u}\cdot\frac{1}{4x+2}\ du = \int \frac{2x+1}{u}\cdot\frac{1}{2(2x+1)}\ du\\ &= \int \frac{1}{u}\cdot\frac{1}{2}\ du = \frac{1}{2}\int\frac{1}{u}\ du\end{aligned}$$

Now we integrate and change back from u to x:

$$\begin{aligned}\int \frac{2x+1}{2x^2+2x+7}\ dx = \frac{1}{2}\int\frac{1}{u}\ du &= \frac{1}{2}\ln(|u|)+K\\ &= \frac{1}{2}\ln\left(|2x^2+2x+7|\right)+K.\end{aligned}$$

This is our final answer.

3 In 2007, there were 420 students in a particular school district. Research indicates that the district expects this number will increase at a rate of $3t + 20$, where t is the number of years since 2007. Estimate the number of students in the district in 2011.

Solution: Let $N(t)$ be the number of students t years from the year 2007. Thus we're asked for $N(4)$, given that $N(0) = 420$ and $N'(t) = 3t + 20$. We simply integrate:

$$N(t) = \int (3t + 20)\ dt = \frac{3}{2}t^2 + 20t + K.$$

In order to find the constant of integration K, we plug in $N(0) = 420$:

$$420 = N(0) = \frac{3}{2}(0)^2 + 20(0) + K = K,$$

so $K = 420$ and $N(t) = \frac{3}{2}t^2 + 20t + 420$. Thus the number of students in 2011 will be $N(4) = \frac{3}{2}(4)^2 + 20(4) + 420 = 524$.

4 The depreciation rate of a new car follows the equation $V'(t) = -5000e^{-0.4t}$ where $V(t)$ is the value of the car and t years after purchase. If the car cost \$22,000 new, what is the value of the car after 4 years?

Solution: We're told that $V'(t) = -5000e^{-0.4t}$ and $V(0) = 22,000$ and asked for $V(4)$. This means we need to find $V(t)$, which we do by integrating:

$$V(t) = \int -5000e^{-0.4t}\ dt = -5000 \int e^{-0.4t}\ dt = -5000 \cdot \frac{1}{-0.4}e^{-0.4t} + K = 12,500e^{-0.4t} + K.$$

To find the constant of integration K, we plug in $t = 0$ and $V(0) = 22,000$:

$$22,000 = V(0) = 12,500e^{-0.4(0)} + K = 12,500 + K,$$

so $K = 22,000 - 12,500 = 9,500$. Thus $V(t) = 12,500e^{-0.4t} + 9,500$ and so $V(4) = 12,500e^{-0.4(4)} + 9,500$, or about \$12,023.71.