

1 (20 points)

- (a) (6 points) What system of equations is represented by the following augmented matrix?

$$\left[\begin{array}{cccc|c} 2 & -2 & 6 & 0 & -18 \\ 0 & 1 & -1 & 0 & 7 \\ 0 & -1 & 1 & 2 & -9 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

- (b) (8 points) Put the above matrix into *reduced* row echelon form. Show the details of your computation, not just the final answer.
- (c) (6 points) What is the solution of the linear system from part (a)?

2 (15 points)

- (a) (6 points) Suppose $A = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 7 & -5 \\ 0 & 3 & -2 \end{bmatrix}$. Find a , b , and c so that $B = \begin{bmatrix} 1 & 1 & a \\ 2 & b & 1 \\ c & 0 & 1 \end{bmatrix}$ is the inverse of A . You must show your work to justify your answer!

Hint: Don't simply calculate A^{-1} ! What properties does A^{-1} have?

- (b) (9 points) Solve the following linear system of equations:

$$\begin{aligned} -y + z &= 2 \\ x + 7y - 5z &= -3 \\ 3y - 2z &= 1 \end{aligned}$$

Hint: Why is this part (b)?

3 (20 points)

- (a) (10 points) While waiting in line at a popular brunch spot, I watch three tables. One group gets 2 orders of eggs and 2 orders of waffles, plus 3 coffees; their bill is \$27.95. Another table has 3 orders of waffles, 1 of eggs, and 4 coffees; their bill is \$28.85. The third table has 4 orders of eggs and 2 coffees; this comes to \$27.90.

Write down a linear system that we could solve to find the prices of eggs, waffles, and coffee. Clearly indicate which variable corresponds to each price. Please **DO NOT SOLVE** this system.

- (b) (10 points) Suppose we have a 100 gallon tank full of water. There are 5 pounds of salt dissolved in the water. Salt water with concentration 0.01 pounds per gallon is being added at a rate of 2 gallons per minute and the (well-stirred) mixture is being poured out at the same rate.

Let $Q(t)$ be the quantity of salt (in pounds) in the tank at time t . Set up an initial value problem that is satisfied by $Q(t)$ and its derivative. **You do not need to solve this equation!**

4 (20 points) Find the general solution of the following differential equations:

(a) (10 points) $y' = \frac{x^2 + 1}{y^2}$

(b) (10 points) $y'' - 4y' + 13y = 0$

5 (25 points) Find the solution of the following initial value problems:

(a) (10 points) $xy' = x^3 + 2y$ where $y(1) = 5$

(b) (15 points) $y'' - 4y' - 12y = 0$ where $y(0) = 1$ and $y'(0) = 4$