

1 (20 points)

- (a) (10 points) Consider the matrix $A = \begin{bmatrix} 1 & 3 & 7 \\ 4 & 1 & 2 \\ 3 & 1 & 2 \end{bmatrix}$. One of the following matrices is actually the inverse matrix A^{-1} . Decide which matrix is actually A^{-1} . Justify your answer with some computations.

$$B = \begin{bmatrix} 5 & -1 & 0 \\ 1 & -2 & 11 \\ -1 & -6 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 4 & -1 & 0 \\ 1 & 2 & 1 \\ -1 & 3 & -12 \end{bmatrix} \quad D = \begin{bmatrix} 0 & 1 & -1 \\ -2 & -19 & 26 \\ 1 & 8 & -11 \end{bmatrix}.$$

- (b) (10 points) Solve the following linear system of equations:

$$\begin{aligned} x + 3y + 7z &= 1 \\ 4x + y + 2z &= -1 \\ 3x + y + 2z &= 2 \end{aligned}$$

2 (20 points)

- (a) (6 points) What system of equations is represented by the following augmented matrix?

$$\left[\begin{array}{cccc|c} 1 & 1 & 3 & 1 & 0 \\ 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 1 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

- (b) (8 points) Put the above matrix into *reduced* row echelon form. Show the details of your computation, not just the final answer.
- (c) (6 points) What is the solution of the linear system from part (a)?

3 (30 points)

- (a) (15 points) Find the solution of $y' - \frac{1}{x}y = x^2$ passing through the point $(x, y) = (1, 1)$.
- (b) (15 points) Find the general solution of $x^2y' + xy = 9y'$.

- 4 (20 points) Twenty (20) pounds of salt are dissolved in a tank containing 50 gallons of water. The water is constantly stirred, distributing the salt evenly throughout the water. Fresh water is pumped into the tank at a rate of 2 gallons per minute, and the well-mixed solution is pumped out at the same rate.
- (a) (15 points) Find an expression for the amount of salt after t minutes.
- (b) (5 points) How long will it take before there is only 5 pounds of salt left in the tank?
- 5 (10 points) Is $y = e^{2x}$ a solution of the differential equation $y'' - 3y' + 2y = 5e^{2x}$? Fully justify your answer.