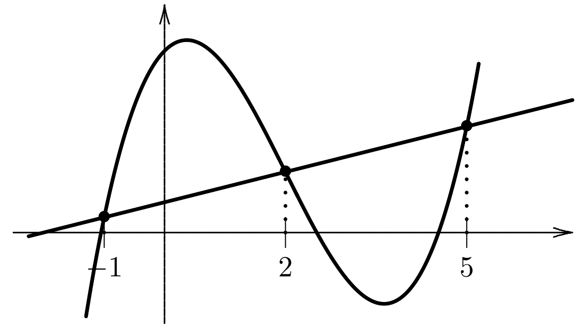
**1** (10 points) Find the area bounded by the curves $y = x + 2$ and $y = x^3 - 6x^2 + 4x + 12$. The curves are graphed to the right. You may assume that the two curves intersect when $x = -1$, when $x = 2$, and when $x = 5$.




**2** (15 points) Let $X$ be a continuous random variable on $0 \le x \le 2$, with probability density function $f(x) = kx^2$.

(a) (5 points) Compute the value of $k$ that makes $f(x) = kx^2$ a probability density function.

(b) (3 points) Compute $P(1 \le X \le 1.5)$

(c) (3 points) Compute $P(X \le 1)$

(d) (4 points) Compute $E(X)$, the expected value of $X$.

**3** (15 points) Doris is waiting for a Circle Link shuttle bus. She's read on the web site that the buses run every 23 minutes. She decides to model the amount of time she must wait as random variable $X$ with a uniform distribution taking values from 0 to 23 minutes.

(a) (5 points) How long should Doris expect to wait for a bus? That is, what is the expected value of $X$?

(b) (5 points) What is the probability that Doris must wait 20 minutes or more for a bus?

(c) (5 points) Doris is running late and will only be on time for her exam if the bus arrives within 3 minutes. What is the probability that a bus will arrive in 3 minutes or less?

**4** (18 points) A mechanic notices something about the cars that she works on. The mileage on the various cars is approximately normally distributed with mean 80 thousand miles and standard deviation 20 thousand miles.

(a) (6 points) What is the probability that a randomly selected car (under the mechanic's care) has between 60 and 100 thousand miles on the odometer?

(b) (6 points) If we say that a car is new if it has less than 15 thousand miles on it, then what percentage of the cars this mechanic works on are new cars?

(c) (6 points) The mechanic overhears someone say that 90% of the cars he works on have less than 150 thousand miles on them. She'd like to compute a similar number – she'd like to be able to say that 90% of the cars she works on have less than $x$ thousand miles. What is $x$?

**5** (20 points) In parts (a) and (b), compute the integrals using any methods from the course:

(a) (6 points) $\int x\cos(6x)\,dx$

(b) (6 points) $\int x^3\sqrt{1+x^2}\,dx$

(c) (8 points) Is it true that $\int_1^\infty \left(\frac{1}{x}\right)^2 dx = \left(\int_1^\infty \frac{1}{x}\,dx\right)^2$? Fully (mathematically) justify your answer.

**6** (10 points) Compute the inverse of each of the following matrices. If the inverse of a matrix does not exist, explain how you know this.

(a) $\begin{bmatrix} 1 & 4 \\ \frac{1}{2} & 2 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ -2\pi & -4\pi & 2\pi+1 \end{bmatrix}$

**7** (15 points) Find all the values of $\alpha$ and $\beta$ so that the system

$$\begin{array}{rcrcrcr} x & + & 3y & - & 2z & = & 5 \\ 2x & + & 4y & + & 3z & = & -3 \\ -x & + & y & + & \alpha z & = & \beta \end{array}$$

(a) ...has a unique solution.

(b) ...has no solution.

(c) ...has infinitely many solutions.

**8** (10 points) A population has a growth rate proportional to its size; that is,

$$\frac{dP}{dt} = kP.$$

Show how to derive the formula $P(t) = P_0 e^{kt}$ from this differential equation.

**9** (15 points) Solve the differential equation $xy' + y = e^x$ with initial condition $y(1) = 2$.

**10** (20 points) A cup of coffee is purchased from McDonald's and brought to the exam, where the room temperature is 70° F. A quick measurement determines that the coffee is 180° F. We let the coffee cool for 5 minutes and find that now the coffee temperature is 175° F. Assume that the rate at which the coffee cools is proportional to the difference between the coffee temperature and the room temperature.

(a) (5 points) Write down the initial value problem describing this situation. That is, write down a differential equation and an initial condition.

(b) (10 points) Find a formula for the coffee temperature as a function of time. Do this by solving the initial value problem in part (a).

(c) (5 points) We'd like to drink the coffee when it is 135° F. How long will it take for the coffee to cool to this temperature?

**11** (12 points) The following table relates the average daily temperature at different portions of a creek with the elevation of that portion of the creek above sea level.

| | | | | |
|---|---|---|---|---|
| Elevation (kilometers): | 2.7 | 2.8 | 3.2 | 3.5 |
| Average Temperature (degrees Celsius): | 11.2 | 10 | 8.5 | 7.5 |

(a) (8 points) Show how to find a line of best fit using the method of least squares for the data in the table above.

(b) (4 points) Use the equation you just found to compute the average daily temperature for this creek at an altitude of 3.2 kilometers.

**12** (25 points) Let $f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$.

(a) (10 points) Find all the first and second partial derivatives $f_x$, $f_y$, $f_{xx}$, $f_{yy}$, $f_{xy}$, and $f_{yx}$ of the function $f(x, y)$.

(b) (10 points) Find all the critical points of the function $f(x, y)$.

(c) (5 points) Classify each of the critical points you found in part (b) as a local maximum, a local minimum, or a saddle point.

**13** (15 points) Find a point on the line $3x + 4y = 5$ that is closest to the point $(0, 0)$ using Lagrange multipliers. Find this minimum distance.
Hint: It is easier to work with the *square* of the distance.