Another Old Final Exam (Spring, 2007)

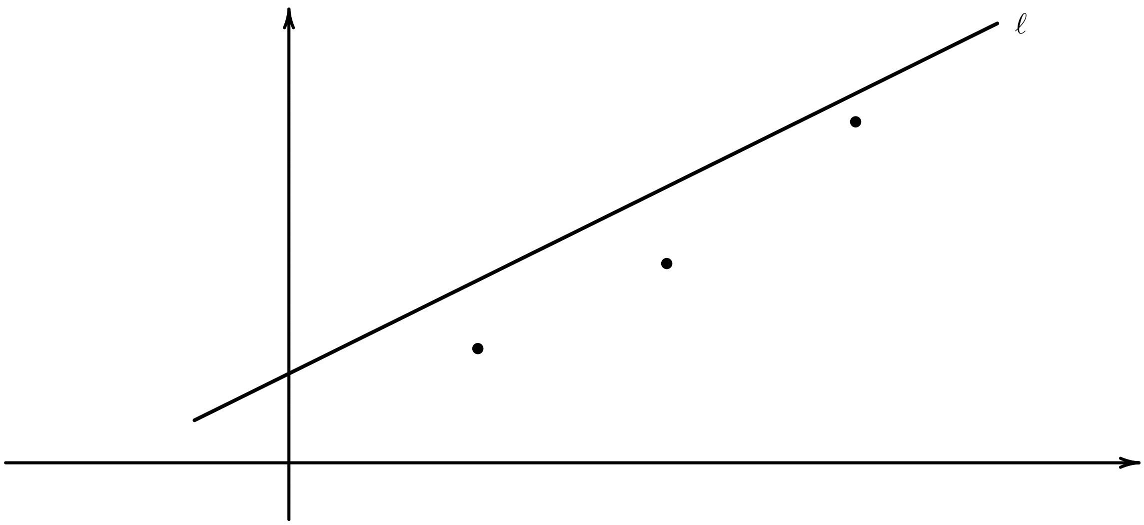
1 (18 points)

(a) (10 points) Find the least squares line (the best fit line) through the following four data points:

$$(1,8), \qquad (3,5), \qquad (5,3), \qquad \text{and} \qquad (7,2).$$

(b) (3 points) Estimate the x-value for which $y = 0$, given the above data.

(c) (5 points) The line shown below (call it line ℓ) is *not* the least squares line (the best fit line) through the three data points shown as dots. Explain why you know this. (The points do *not* all lie on a single line.)



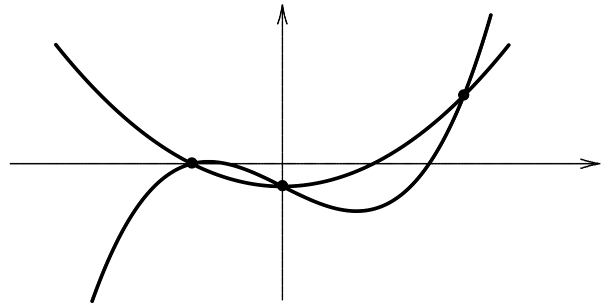
2 (18 points) Compute the following integrals. Show your work in order to justify your answer. For improper integrals, you *must* include limits in your justification.

(a) (6 points) $\displaystyle\int xe^{2x}\,dx$

(b) (6 points) $\displaystyle\int x\sqrt{1+x}\,dx$

(c) (6 points) $\displaystyle\int_0^4 \frac{1}{\sqrt{x}}\,dx$

3 (10 points) Find the area bounded by the curves $y = x^3 - 2x - 1$ and $y = x^2 - 1$. The curves are graphed to the right.



4 (12 points)

(a) (6 points) Find k (if possible) so that $f(x) = \frac{k}{x^3}$ is a probability density function on the interval $x \geq 2$ (that is, on the interval $2 \leq x < \infty$).

(b) (6 points) Suppose X is the random variable with the probability density function from part (a) (with the k you found). Find $P(X \geq 3)$, the probability that X is at least 3.

5 (15 points) Suppose the life span of an automobile oil filter is approximately normally distributed with mean $\mu = 6,000$ miles and standard deviation $\sigma = 1,000$ miles.

(a) (6 points) Find the probability that a filter lasts $8,000$ miles or less.

(b) (9 points) Suppose the manufacturer wants to guarantee their product by offering full refunds if the oil filter fails within a certain number of miles. The company only wants to refund at most 1% of their sales. For how many miles should they warrant their oil filters? Round your answer to the nearest multiple of 25 miles.

6 (12 points) A refrigerator comes with a full two-year warranty. The manufacturer has found that the time before this model of refrigerator malfunctions (in some way) is approximately an exponential random variable with mean 4 years.

(a) (5 points) Find the exponential density function $f(x)$ for this random variable.

(b) (7 points) What percentage of refrigerators will experience some sort of malfunction during the (two-year) warranty period?

7 (15 points) Suppose an urn contains twelve plastic chips: five red and seven yellow. A chip is pulled from the urn, its color is noted, and then it is returned to the urn. This is repeated five times.

(a) (5 points) What is the probability that exactly three of the five chips drawn from the urn are yellow?

(b) (5 points) What is the probability that at least four of the five chips drawn from the urn are yellow?

(c) (5 points) Find the expected number of red chips seen in the five draws.

8 (19 points) Solve the system

$$\begin{aligned} x_1 - x_2 + 2x_3 + 3x_4 &= 2 \\ 2x_1 + 2x_2 + x_4 &= 8 \\ -x_1 + x_2 + 3x_3 + 2x_4 &= 3 \end{aligned}$$

in the following way.

(a) (5 points) Write this system as an augmented matrix $[\ A \mid B\ ]$.

(b) (10 points) Use row operations to put your augmented matrix in *reduced* row echelon form. Be sure to show your steps!

(c) (4 points) Write down the solution of your system in a form that does not involve matrices.

9 (18 points) A dental tool has been sterilized using heat. It has a temperature of 125 C when it is taken out of an autoclave and placed in a cooling chamber. The temperature of the cooling chamber is 20 C. The rate of change of the temperature $f(t)$ of the dental tool after t minutes is proportional to the difference of the temperatures of the tool and the cooling chamber.

(a) (6 points) Set up an initial value problem that is satisfied by the temperature $f(t)$ of the cooling dental tool, as described above.

(b) (2 points) Determine the sign of the constant of proportionality in your differential equation. Justify your answer with appropriate reasoning.

(c) (10 points) Solve the following initial value problem:

$$xy' + 3y = \frac{2}{x}e^{x^2}$$
$$y(1) = 0$$

10 (18 points) Solve the following differential equations or initial value problems.

(a) (6 points) $y'' - 8y' - 12y = 0$

(b) (6 points) $y'' - 4y' + 13y = 0$ where $y(0) = 1$ and $y'(0) = 4$

(c) (6 points) Find the constant b so that $y = 5te^{4t}$ is a solution of the differential equation $y'' + by' + 16y = 0$.

11 (15 points) Find the critical points of the function

$$f(x, y) = 2x^2 - 3xy + y^2 + 3x - y + 11$$

and classify them (if possible) as local maxima, local minima, or saddle points. Compute all necessary partial derivatives to justify your answer.

12 (15 points) Using the method of Lagrange multipliers, find the minimum distance between the origin and the curve $xy^2 = 54$ and the point (or points) at which this minimum occurs.

13 (15 points) We are building a rectangular box (with no lid) that we would like to have volume of 108 cm^3. What is the smallest amount of material needed? (You may assume that "amount of material" means the surface area.)