

Your Name

Your Signature

Lecture: 8:30
 9:30

SI: Alex Avni Jason
 Ashley Becca Matt

| Problem | Total Points | Score |
|---------|--------------|-------|
| 1 | 25 | |
| 2 | 30 | |
| 3 | 35 | |
| 4 | 30 | |
| 5 | 30 | |
| Total | 150 | |

- You are only allowed to use the attached formula sheet; no other notes are allowed for this exam.
- You may use a calculator, such as a scientific or graphing calculator. You may not use your cellular phone. You are not allowed to share calculators.
- In order to receive partial credit, you must show your work. Be wary of doing computations in your head. Instead, write out your computations on the exam paper.
- Your answers should either be *exact answers* (like $2\sqrt{2}$) or rounded to four digits after the decimal (like 0.1234) in whatever units you're using. Be careful not to round intermediate calculations whenever possible, as you will lose points for errors due to incorrect rounding.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- Good luck!

1 (25 points) Consider a population P satisfying the differential equation $\frac{dP}{dt} = 5P^2 - 200P$.

(a) (10 points) Identify the equilibrium solutions of the differential equation. (That is, for what values of P will the population be constant?)

(b) (8 points) For what values of P is the population increasing?

(c) (7 points) Describe what happens to the population as $t \rightarrow \infty$ if the initial population is $P(0) = 10$.

2 (30 points) A large tank is filled to capacity with 500 gallons of pure water. Brine containing 2 pounds of salt per gallon is pumped into the tank at a rate of 5 gal/min. The well-mixed solution is pumped out at the same rate.

(a) (25 points) Find an expression for the amount of salt in the tank after t minutes.

(b) (5 points) In the long run, how much salt should we expect to have in the vat?

3 (35 points) Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) (20 points) Use elimination to find A^{-1} .

(b) (15 points) Find constants a , b , and c so that the parabola given by $y = ax^2 + bx + c$ passes through the points $(1, 1)$, $(-1, -5)$, and $(0, -4)$.

4 (30 points) Consider the system of equations below:

$$\begin{aligned}x_1 + 2x_2 + x_3 + 2x_4 - 3x_5 &= -3 \\x_1 - x_2 + 4x_3 + 2x_4 + 9x_5 &= 6 \\2x_1 + x_2 + 5x_3 + 5x_4 + 3x_5 &= 1\end{aligned}$$

- (a) (20 points) Write the augmented matrix for the system of equations and use row elimination to put it in reduced row-echelon form. Show the details of your computation, not just the final answer.

- (b) (10 points) What is the solution of this linear system?

5 (30 points)

- (a) (10 points) Find the solution to the initial value problem

$$y' = e^x y + xy, \quad y(0) = 3.$$

- (b) (10 points) Find the general solution to the differential equation

$$y'' - 4y' - 5y = 0.$$

- (c) (10 points) Find the solution to the initial value problem

$$y'' - 4y' + 5y = 0, \quad y(0) = 2, \quad y'(0) = 1.$$

Note: This is *not* the differential equation from part (b).