Inverses of Matrices

Computing Inverses

The inverse of a 2×2 matrix:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \Longrightarrow \qquad A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

If $ad - bc = 0$, then A has no inverse.

The inverse of a general $n \times n$ matrix:
Perform row reduction to change the extremely augmented matrix $[\,A \mid I\,]$ into the form $[\,I \mid A^{-1}\,]$. You can read off A^{-1} from this. (If you can't reach I on the left-hand side of the vertical bar, then there is no inverse of A.)

Compute the inverses (if they exist) of the following matrices:

1. $A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$

2. $B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

3. $C = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

4. $D = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$

5. $E = \begin{bmatrix} -2 & 4 \\ 1 & 2 \end{bmatrix}$

6. $F = \begin{bmatrix} -2 & 4 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}$

7. $G = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

8. $H = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$

9. $J = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 1 \\ 2 & 2 & -2 \end{bmatrix}$

Answers:

1. $A^{-1} = A$
2. $B^{-1} = \begin{bmatrix} 0 & 1/2 \\ 1 & -1/2 \end{bmatrix}$
3. $C^{-1} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
4. D^{-1} DNE
5. $E^{-1} = \begin{bmatrix} -1/4 & 1/2 \\ 1/8 & 1/4 \end{bmatrix}$
6. F^{-1} DNE
7. $G^{-1} = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$
8. $H^{-1} = \begin{bmatrix} 1/2 & -1/2 & -1 \\ 1 & -2 & -1 \\ -1/2 & 3/2 & 1 \end{bmatrix}$
9. J^{-1} DNE