# Local Maxima & Minima



## Tests for Maxima, Minima, Saddle Points

**Critical Points:**

A point $(x_0, y_0)$ is a *critical point* for the function $z = f(x, y)$ if $f_x(x_0, y_0) = 0$ and $f_y(x_0, y_0) = 0$.

That is, critical points are those points where $f_x = 0$ and $f_y = 0$.

**Is A Critical Point A Max, Min, or Saddle?**
Set $D = f_{xx}f_{yy} - f_{xy}^2$.

- $D < 0$ at $(x_0, y_0) \implies (x_0, y_0)$ is a saddle point.
- $D > 0$ at $(x_0, y_0)$
  - $f_{xx} < 0$ at $(x_0, y_0) \implies (x_0, y_0)$ a local max
  - $f_{xx} > 0$ at $(x_0, y_0) \implies (x_0, y_0)$ a local min
- $D = 0$ at $(x_0, y_0) \implies$ no info about $(x_0, y_0)$

For each of the following functions...

(a) Compute $f_x$, $f_y$, $f_{xx}$, $f_{yy}$, and $f_{xy}$. (You can assume that $f_{yx} = f_{xy}$.)

(b) Find the critical points. That is, find all points where both $f_x = 0$ and $f_y = 0$.

(c) Find the value of $f_{xx}$ and $D$ at each critical point.

(d) Using the test above, determine whether each critical point is a local maximum, a local minimum, or a saddle point.

**1** $f(x, y) = x^2 - y^2$

**2** $f(x, y) = x^2 + 4y^2 - 2xy$

**3** $f(x, y) = x^2 + 2xy + 2y^2 - 8y + 12$

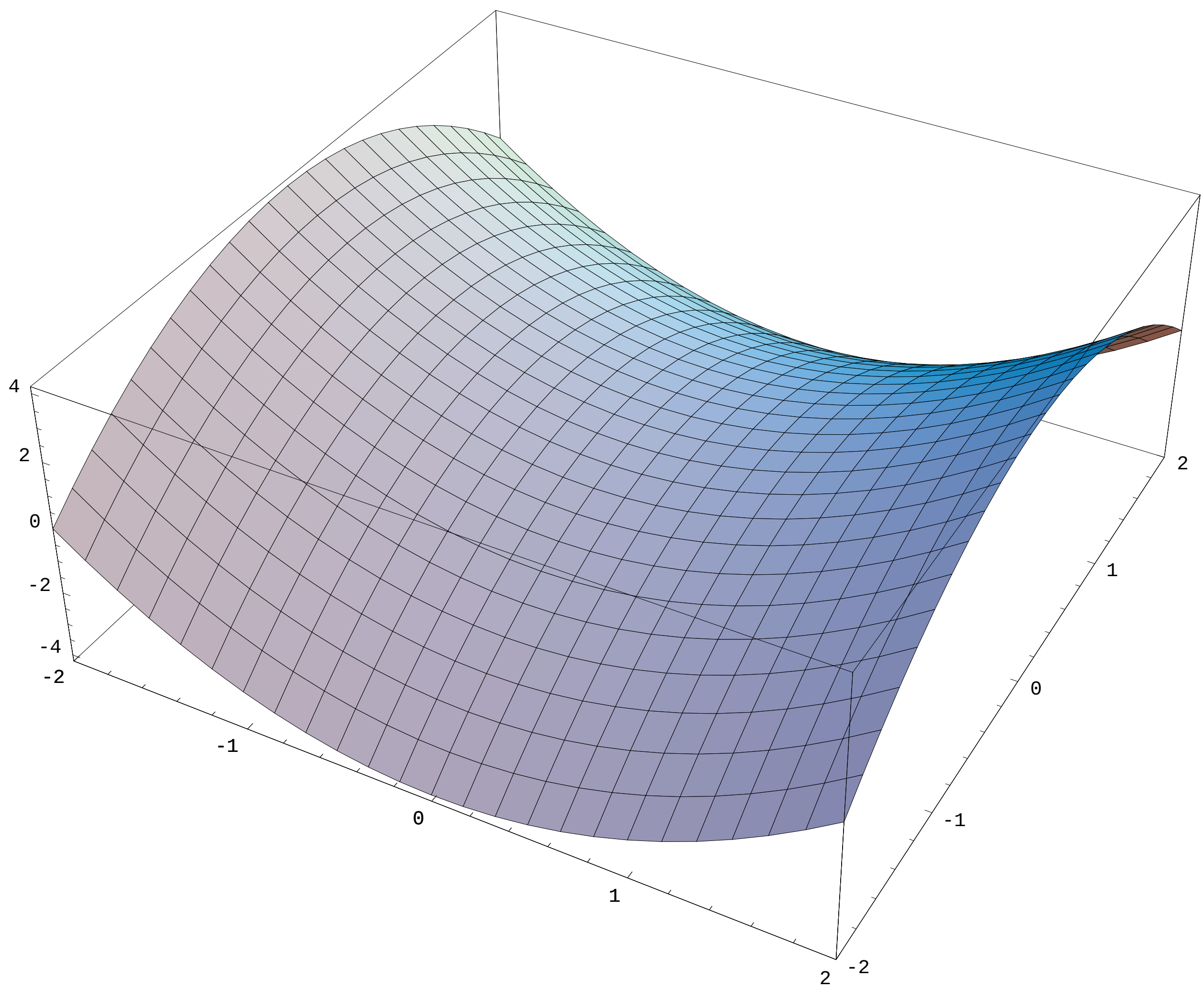
**4** $f(x, y) = x^2 - 2y^2 + xy - 4$

**5** $f(x, y) = y^2 + 2xy - x^2 + 2x - 2y + 3$

**6** $f(x, y) = 8 - x^2 - xy - y^2$

**7** $f(x, y) = \dfrac{x}{x + y}$

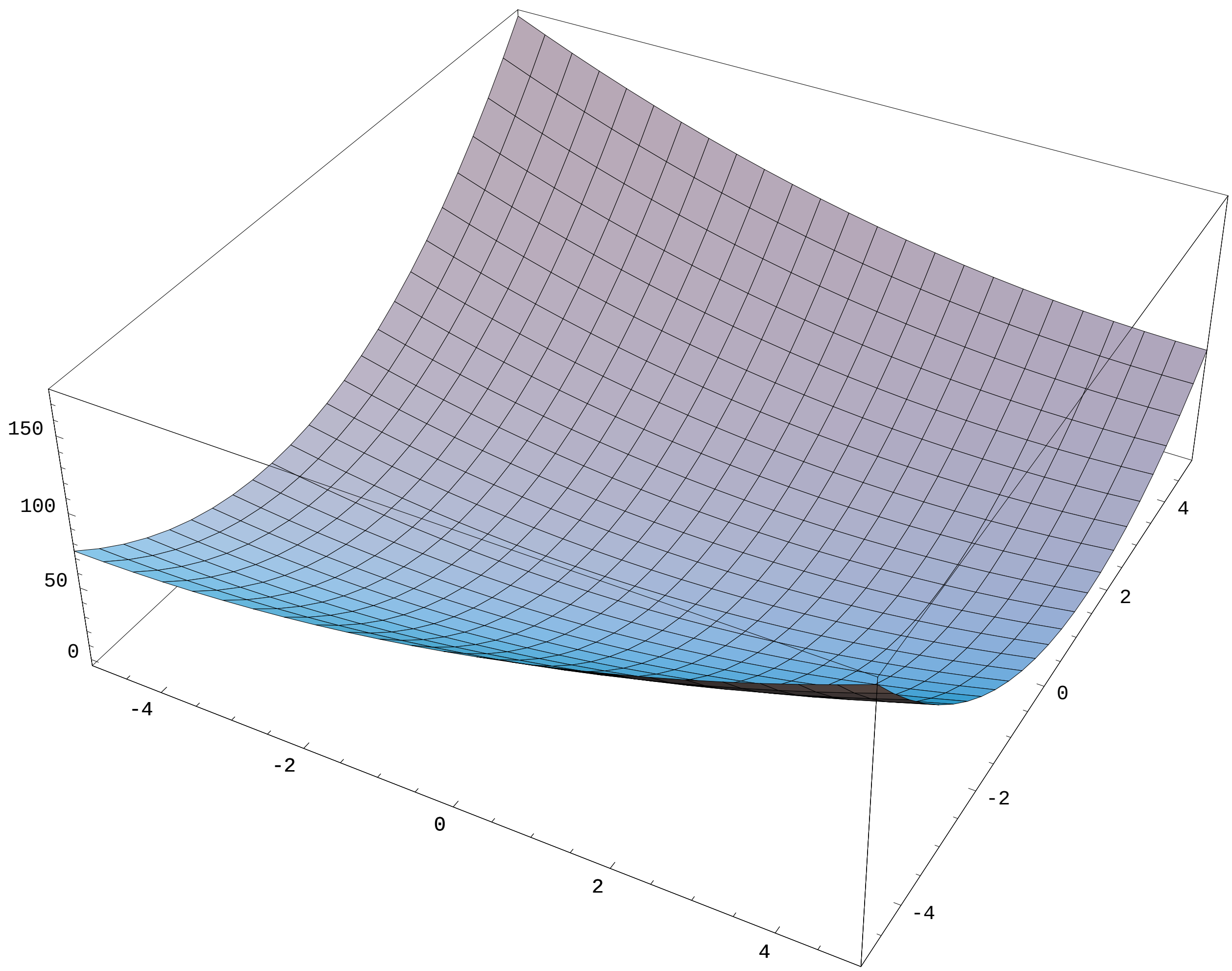
**8** $f(x, y) = x^4 + y^4$




1

$$z = f(x, y) = x^2 - y^2$$

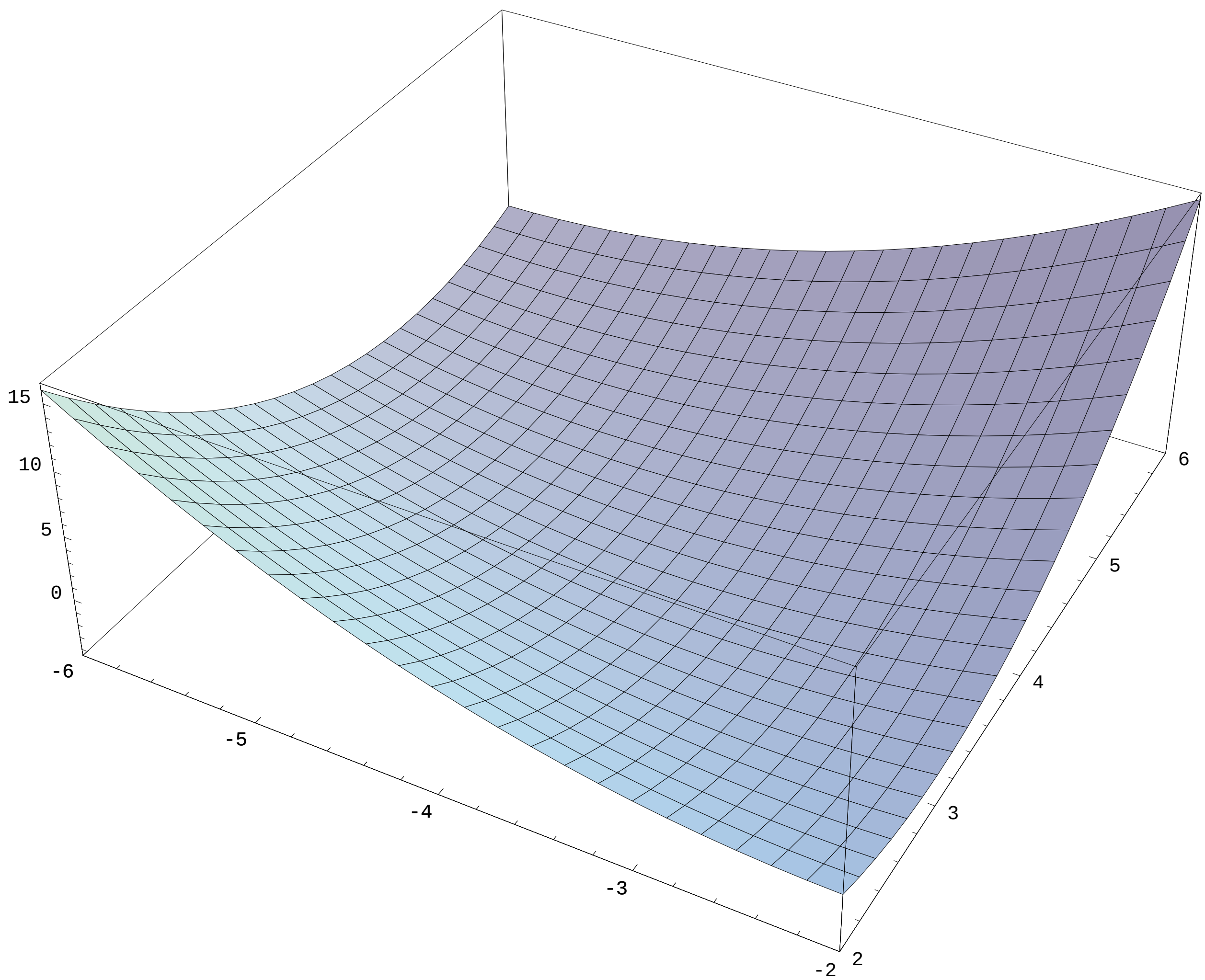
Saddle point at $(x, y) = (0, 0)$




2

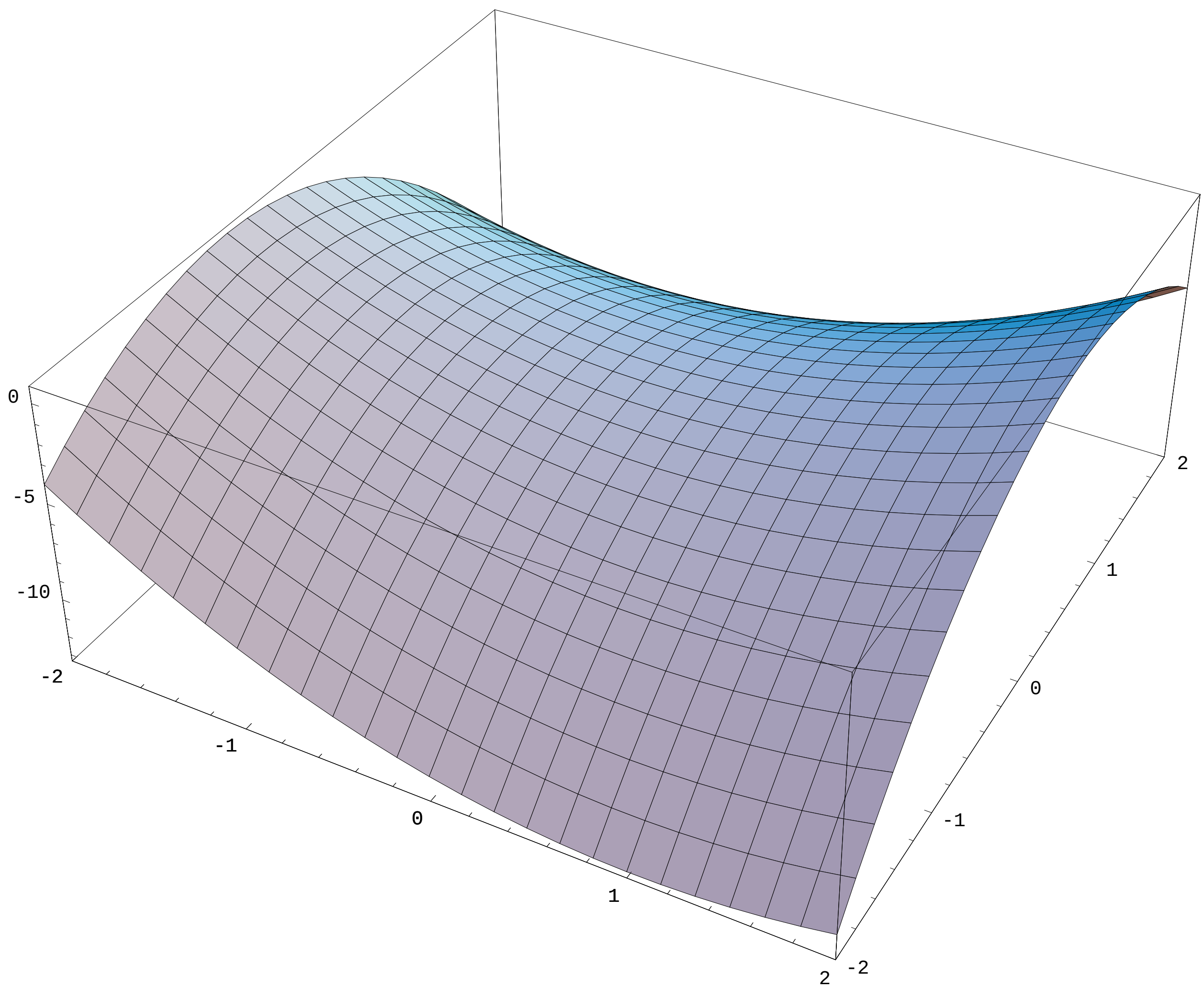
$$z = f(x, y) = x^2 + 4y^2 - 2xy$$

Local minimum at $(x, y) = (0, 0)$



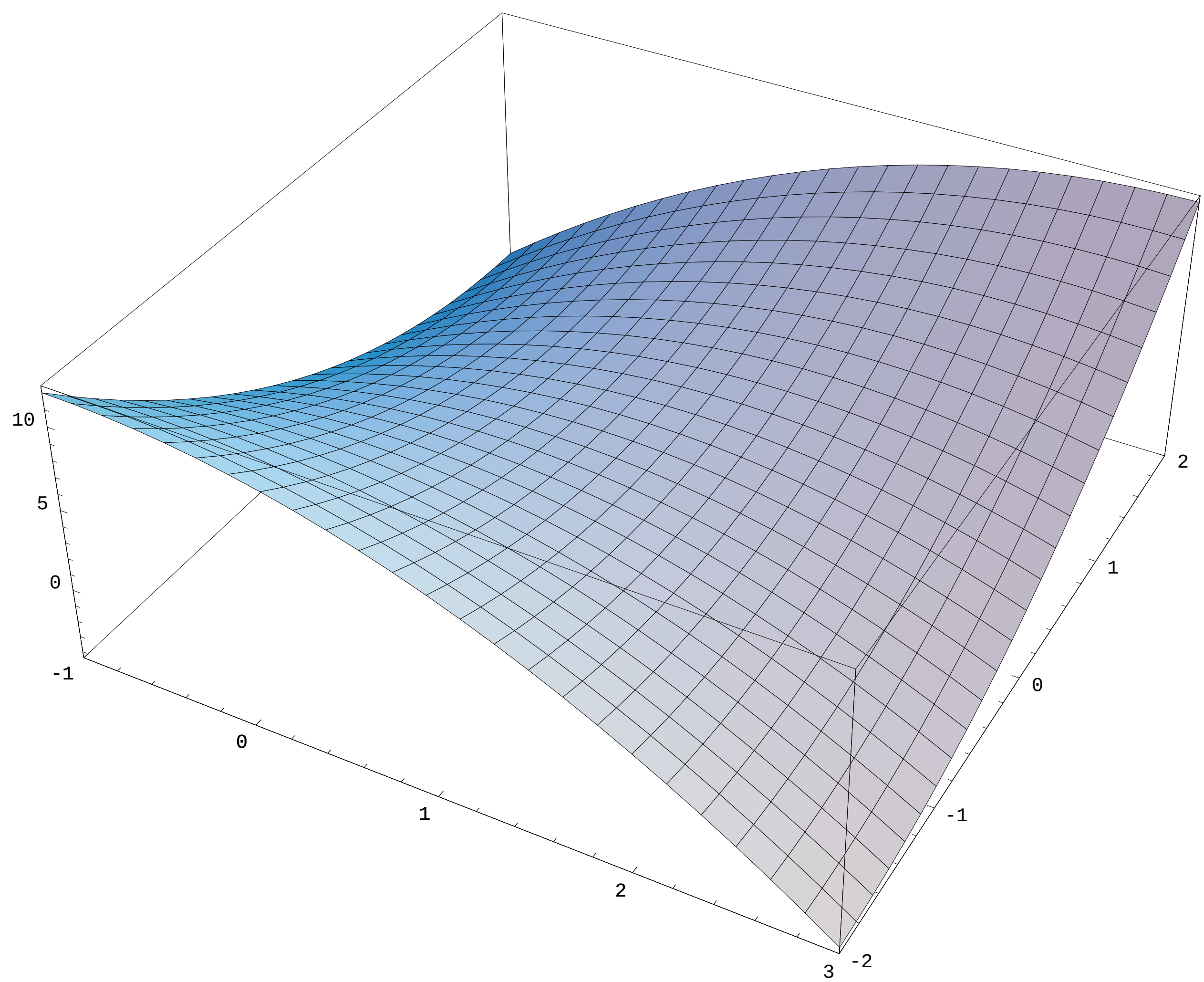

3 $$z = f(x,y) = x^2 + 2xy + 2y^2 - 8y + 12$$

Local minimum at $(x, y) = (-4, 4)$



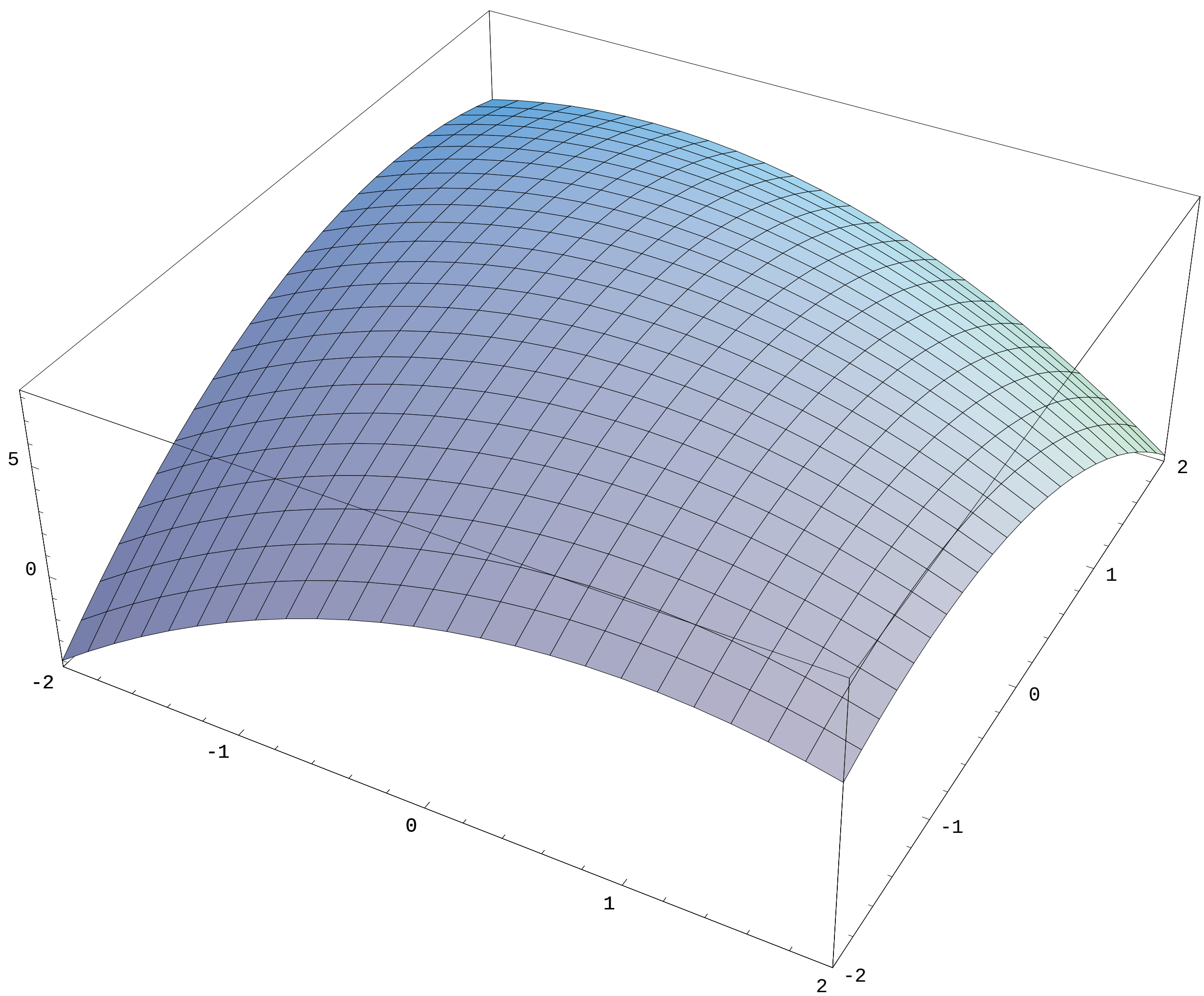

$\boxed{4}$ $$z = f(x, y) = x^2 - 2y^2 + xy - 4$$

Saddle point at $(x, y) = (0, 0)$



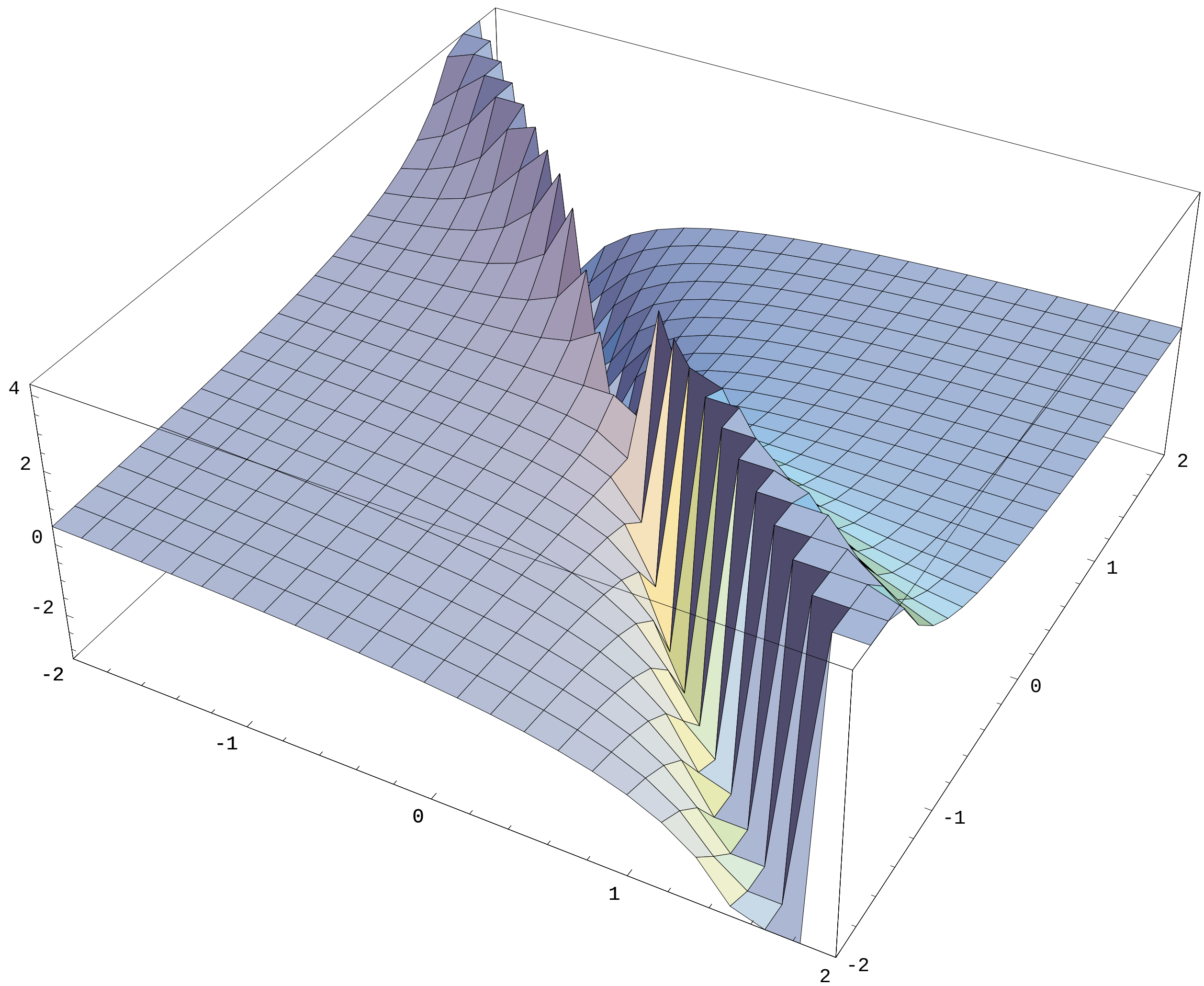

5 $$z = f(x,y) = y^2 + 2xy - x^2 + 2x - 2y + 3$$

Saddle point at $(x, y) = (1, 0)$




6 $$z = f(x, y) = 8 - x^2 - xy - y^2$$

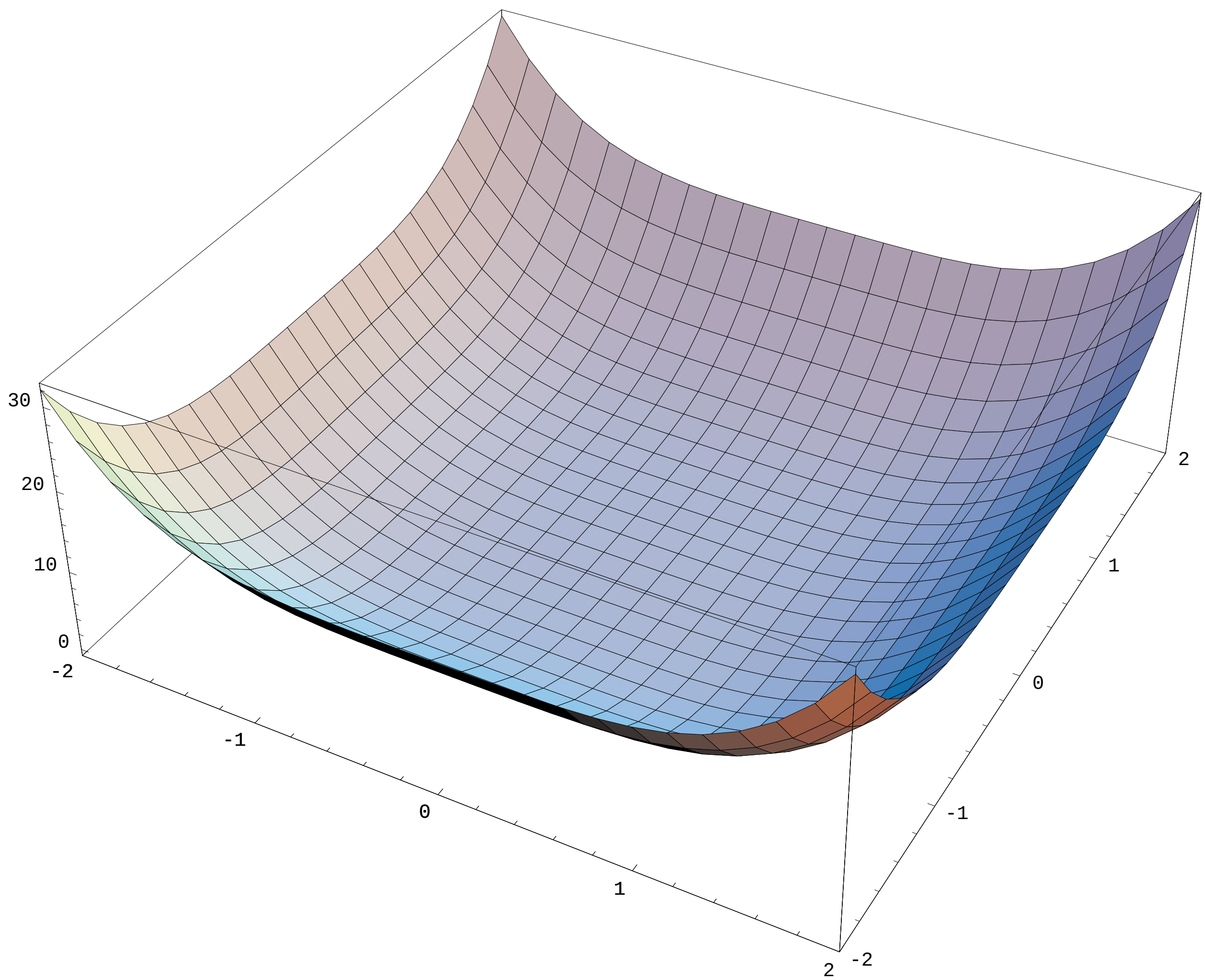
Local maximum at $(x, y) = (0, 0)$




7 $$z = f(x,y) = \frac{x}{x+y}$$

No critical point – $(0,0)$ is not in the domain

(The domain is all $(x,y)$ not on the line $x+y=0$.)




8 $$f(x, y) = x^4 + y^4$$

Local minimum at $(x, y) = (0, 0)$

However, our test fails.