1 A company is making two products. The demand functions are given by

$$p_1 = 26 - 2x \qquad \text{and} \qquad p_2 = 18 - y$$

where $x$ and $y$ are given in thousands of units and the prices $p_1$ and $p_2$ are in thousands of dollars. The joint cost function is given by $C(x, y) = 2x^2 + 4xy + y^2$. Find the maximum possible profit.

**Hint:** Recall that profit is revenue minus cost: $P = R - C$.

2 In the Canadian postal system, a regular parcel may not have the length plus girth exceed 300 cm. What are the dimensions of the largest (in terms of volume) rectangular box that can be shipped as a regular parcel.

**Hint:** Write the volume in terms of three variables. Use the "length plus girth" condition to solve for one of the variables, then write the volume in terms of the other two.

3 A company makes two kinds of bottled water. The sparkling water costs \$2 a bottle to produce, and the uncarbonated costs \$1. A study of the market determines that if the sparkling sells for \$$s$ per bottle and the plain sells for \$$p$ per bottle, they will sell $n = 12 - 10s + 7p$ bottles of sparkling and $m = 15 + 8s - 9p$ bottles of the uncarbonated each day. In this problem we'll find the prices that maximize the company's profit.

(a) How much does it cost to produce $n = 12 - 10s + 7p$ bottles of sparkling and $m = 15 + 8s - 9p$ bottles of uncarbonated water?

(b) Find the revenue from selling $n = 12 - 10s + 7p$ bottles of sparkling and $m = 15 + 8s - 9p$ bottles of uncarbonated water.

(c) Use your answers for parts (a) and (b) to write down a function for the profit from selling $n = 12 - 10s + 7p$ bottles of sparkling and $m = 15 + 8s - 9p$ bottles of uncarbonated water.

(d) Maximize the function you found in part (c).

4 We are building a rectangular box (with no lid) that we would like to have volume of 256 in$^3$. What is the smallest amount of material needed. (You may assume that "amount of material" means the surface area.)

**Answers:**

1 $(x, y) = (2, 2.5)$ 2 $\ell = 100$ cm, $w = h = 50$ cm

3 $s \approx \$4.31$, $p \approx \$4.15$ 4 $\ell = w = 8$ in, $h = 4$ in