

## Reduced Row Echelon Form

### Example:

$$\left[ \begin{array}{ccccc|c} 1 & 0 & 3 & 0 & 1 & 6 \\ 0 & 1 & 2 & 0 & 0 & 5 \\ 0 & 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

augmented matrix

corresponds  
to the system

$$\begin{aligned} x_1 + 3x_3 + x_5 &= 6 \\ x_2 + 2x_3 &= 5 \\ x_4 + 2x_5 &= 2 \\ 0 &= 1 \\ 0 &= 0 \end{aligned}$$

corresponding system

### General Conditions:

1. First non-zero term in each row (the “pivot”) is a 1; there are zeros above and below it.
2. The pivot in any row is to the right of the pivot in the row above it.
3. Any rows containing all zeros (to the left of the vertical bar) appears at the bottom.

For each of the following problems...

- (a) Determine whether or not the matrix given is in reduced row echelon form. If it is not, use elementary row operations to put it in reduced row echelon form.
- (b) Determine whether the system represented by the matrix has solutions (is *consistent*) or has no solutions (is *inconsistent*).
- (c) If the system is consistent, write down the solutions to the system.

$$\boxed{1} \quad \left[ \begin{array}{cc|c} 1 & 1 & 12 \\ 0 & 1 & 17 \end{array} \right]$$

$$\boxed{2} \quad \left[ \begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 0 & 5 \end{array} \right]$$

$$\boxed{3} \quad \left[ \begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & 5 & 2 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\boxed{4} \quad \left[ \begin{array}{ccc|c} 1 & -1 & 3 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

$$\boxed{5} \quad \left[ \begin{array}{ccccc|c} 1 & 1 & 0 & 2 & 0 & 3 \\ 0 & 0 & 1 & -2 & 0 & 4 \\ 0 & 0 & 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right]$$

$$\boxed{6} \quad \left[ \begin{array}{ccccc|c} 1 & 0 & -1 & 0 & 1 & 7 \\ 0 & 1 & 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 1 & 2 & 2 \\ 0 & 0 & 0 & 2 & 4 & 1 \end{array} \right]$$

For each of the following systems,...

- (a) Write down the augmented matrix.
- (b) Use elimination to put the augmented matrix in reduced row echelon form.
- (c) If possible, solve the system by reading the solution from the reduced row echelon form.

$$\begin{array}{l}
 \boxed{7} \quad x_1 - 2x_2 + 4x_3 = 6 \\
 -2x_1 + 3x_2 - 6x_3 = -14 \\
 3x_1 + x_2 - x_3 = -1
 \end{array}$$

$$\begin{array}{l}
 \boxed{8} \quad x_1 + 2x_2 + 4x_3 = 7 \\
 2x_1 + x_2 - 7x_3 = 5 \\
 x_2 + 4x_3 = 3
 \end{array}$$

$$\begin{array}{l}
 \boxed{9} \quad x_1 + x_2 + x_3 = 3 \\
 x_1 + 3x_3 = 0 \\
 2x_1 + 2x_2 + 1x_3 = 5
 \end{array}$$

$$\begin{array}{l}
 \boxed{10} \quad x_1 + x_2 + x_3 + x_4 = 6 \\
 2x_1 + 3x_2 = -1 \\
 3x_1 + 3x_2 - 1x_3 - 1x_4 = -1
 \end{array}$$

$(x, y) = (-5, 17)$	$\boxed{2}$	inconsistent
$(x, y, z) = (z, 5z - 2, z)$	$\boxed{4}$	$(x, y, z) = (z, 5z - 2, z)$
$(x, y, z) = (z, 5z - 2, z)$	$\boxed{5}$	$(x, y, z) = (z, 5z - 2, z)$
$(x, y, z) = (z, 5z - 2, z)$	$\boxed{7}$	inconsistent
$(x, y, z) = (z, 5z - 2, z)$	$\boxed{6}$	$(x, y, z) = (z, 5z - 2, z)$
$(x, y, z) = (z, 5z - 2, z)$	$\boxed{8}$	$(x, y, z) = (z, 5z - 2, z)$
$(x, y, z) = (z, 5z - 2, z)$	$\boxed{10}$	$(x, y, z) = (z, 5z - 2, z)$

**Answers:**