MATH 126 — Spring, 2008

Lagrange Multipliers

Finding Local Maxima/Minima Using Lagrange Multipliers

1. Identify the function $f(x, y)$ you wish to maximize (or minimize).
2. Identify the constraint equation $g(x, y) = 0$.
3. Form the new function $F(x, y, \lambda) = f(x, y) + \lambda g(x, y)$.
4. Solve the equations

$$\begin{aligned} F_x &= 0 & & & f_x(x,y) &= -\lambda g_x(x,y) \\ F_y &= 0 & &\text{or} & f_y(x,y) &= -\lambda g_y(x,y) \\ F_\lambda &= 0 & & & g(x,y) &= 0 \end{aligned}$$

5. Identify the points you found in step 4 as local maxima, local minima, or neither.

1 Find the minimum value of $x^2 + 2y^2$ subject to the constraint that the points (x, y) lie on the line $4x - 3y = 41$.

2 Maximize the function $f(x, y) = xy$ subject to the constraint that the points (x, y) lie on the ellipse $9x^2 + 16y^2 = 144$.

3 Find the smallest distance between the origin and the curve $x^2y = 16$.

4 Find the smallest value of $f(x, y, z) = x^2 + 2y^2 + 3z^2$ given that $x + y + z = 11$.

Answers:

1 Minimum at $(x, y) = (8, -3)$

2 Two maxes: $(2\sqrt{2}, 3/\sqrt{2})$ and $(-2\sqrt{2}, -3/\sqrt{2})$;
Two mins: $(-2\sqrt{2}, 3/\sqrt{2})$ and $(2\sqrt{2}, -3/\sqrt{2})$

3 $(x, y) = (\pm 2\sqrt{2}, 2)$, so distance is $\sqrt{12} = 2\sqrt{3}$

4 Minimum is $f(6, 3, 2) = 66$