Lagrange Multipliers II

1 Maximize $f(x,y) = x + y + z$ subject to the constraint $x^2 + y^2 + z^2 = 27$.

For each problem below,

(a) Graph the constraint equation $g(x,y) = 0$.

(b) Graph "level curves" of the function $f(x,y)$ that you are maximizing / minimizing. That is, graph $f(x,y) = k$ for various constants k: $f(x,y) = 0$, $f(x,y) = 1$, and so on.

(c) From your graphs, approximate the maximum and minimum values of $f(x,y)$ subject to the constraint $g(x,y) = 0$.

(d) Find the exact maximum and minimum value(s) using the procedure from Wednesday.

2 Find the minimum value of $x^2 + y^2$ subject to the constraint that the points (x,y) lie on the line $x + y = 8$.

3 Maximize (and minimize) the function $f(x,y) = x - y$ subject to the constraint that the points (x,y) lie on the ellipse $x^2 + 4y^2 = 36$.

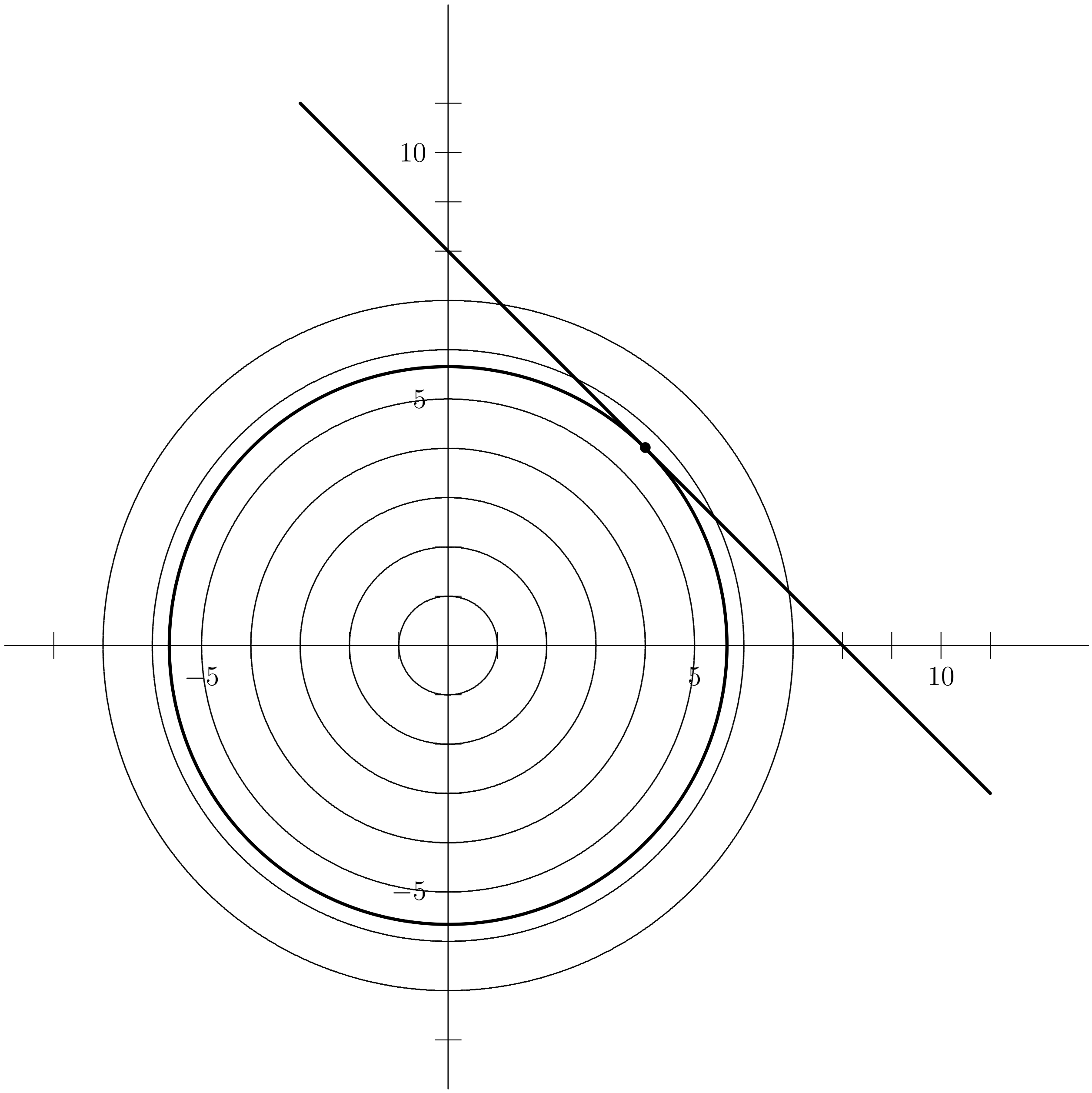
4 Find the smallest distance between the origin and the curve $xy^2 = 2$.

Answers:

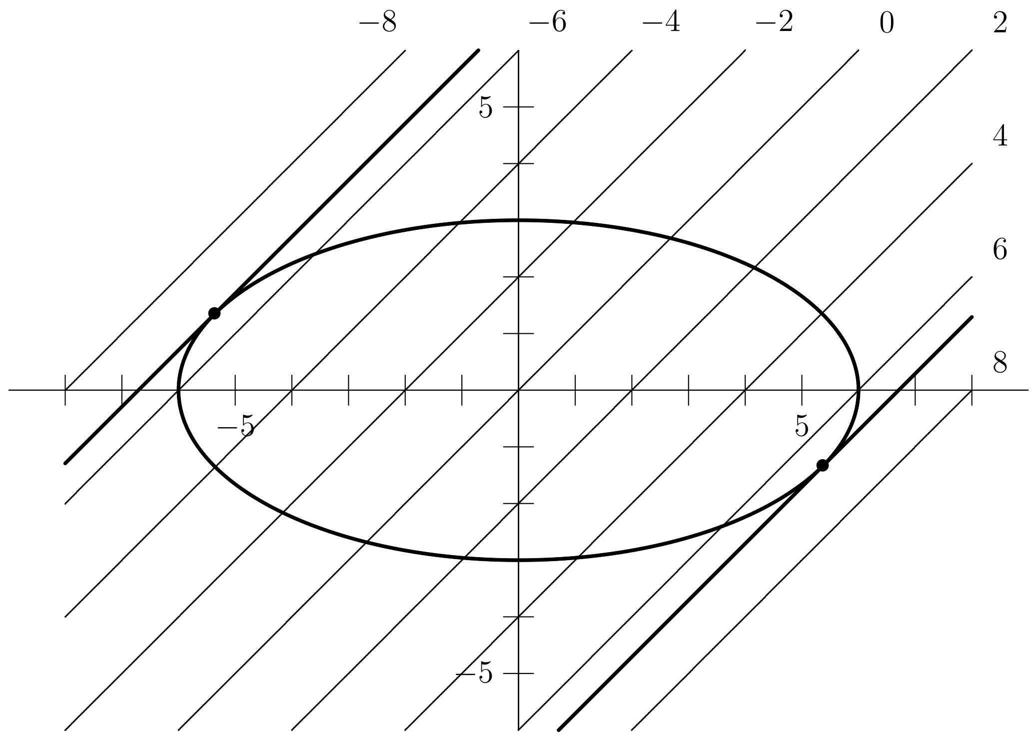
1 $f(3,3,3) = 9$ **2** $4^2 + 4^2 = 32$

3 Max: $(^{12}/_{\sqrt{5}}, -^{3}/_{\sqrt{5}})$ Min: $(-^{12}/_{\sqrt{5}}, ^{3}/_{\sqrt{5}})$

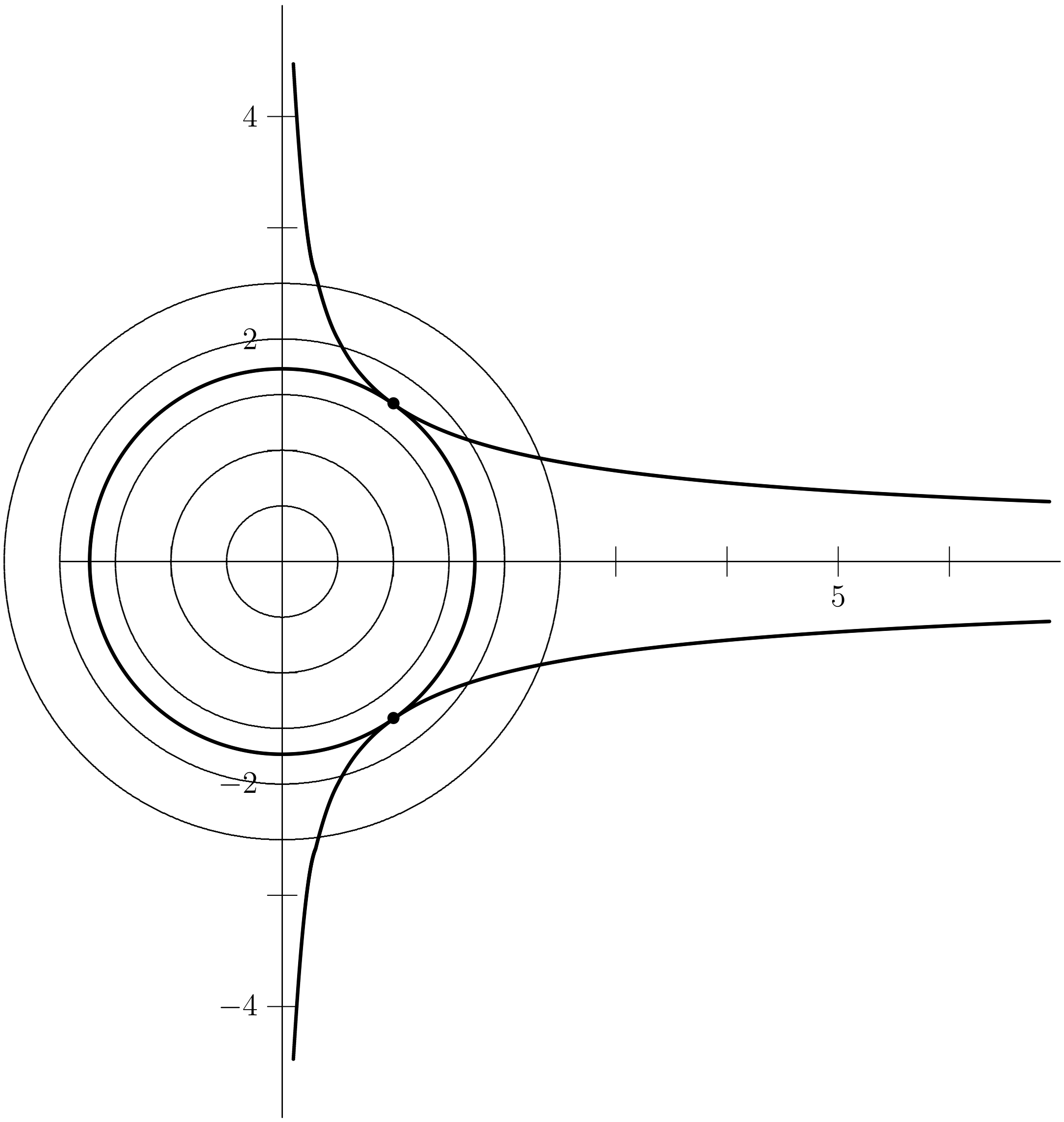
4 $(x,y) = (1, \pm\sqrt{2})$, so distance is $\sqrt{3}$



2 The line $x + y = 8$ together with a number of circles $x^2 + y^2 = k$. The smallest circle that touches the line is the one that is tangent at $(x, y) = (4, 4)$, so $x^2 + y^2 = 32$.



3 The ellipse $x^2+4y^2 = 36$ together with a number of lines $x-y = k$. I've written the k value at the upper right corner of each line. The smallest value of $x-y$ is thus clearly between -6 and -8, and the largest is between 6 and 8. The actual largest occurs at $(x, y) = ({}^{12}\!/\!_{\sqrt{5}}, -{}^{3}\!/\!_{\sqrt{5}})$, where $x - y = {}^{15}\!/\!_{\sqrt{5}} = 3\sqrt{5} \approx 6.7082$. The smallest is at $(x, y) = (-{}^{12}\!/\!_{\sqrt{5}}, {}^{3}\!/\!_{\sqrt{5}})$, where $x - y = -{}^{15}\!/\!_{\sqrt{5}} = -3\sqrt{5} \approx -6.7082$.



4 The curve $xy^2 = 2$ together with circles of varying radii centered at the origin. The points closest to the origin are those points on the smallest circle. This smallest circle is shown; it has radius $r = \sqrt{3} \approx 1.7321$ and hits the curve at $(x, y) = (1, \pm\sqrt{2}) \approx (1, \pm 1.4142)$.