



1 We create a table summarizing the expected profit for 10, 11, 12, 13, and 14 cars. This table is:

Number of Cars	10	11	12	13	14
Expected Profit	\$150.00	\$162.60	\$170.40	\$172.20	\$166.80

Thus, to maximize profit, we should have 13 cars on hand.

Here’s the computation for 12 cars: We rent 10 cars with a probability of 0.1, 11 cars with a probability of 0.2, and 12 cars with a probability of  $0.25 + 0.3 + 0.15 = 0.7$ . When we rent 10 cars, we have a profit of  $10 \times \$15 = \$150$  plus a loss of  $2 \times \$9 = \$18$  (for the two un-rented cars). Thus, with probability 0.1, we have a profit of  $\$150 - \$18 = \$132$ . That is, we’ve made the following chart:

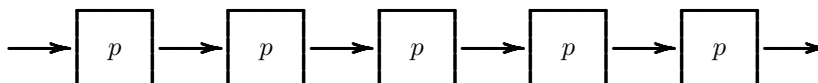
Number of Customers	10	11	12	13	14
Number of Cars Rented	10	11	12		
Profit	\$132	\$156	\$180		
Probability	0.1	0.2	$0.25 + 0.3 + 0.15 = 0.70$		

Continuing on, we get an expected profit of

$$(0.10)(\$132) + (0.2)(\$156) + (0.70)(\$180) = \$170.40$$

when we have 12 cars available. The other computations are similar.

2 As an example, we could test five at once:



Then the expected number of tests per component is

$$\frac{1 \cdot P(\text{ all pass } ) + 6 \cdot P(\text{ at least one fails } )}{5} = \frac{1 \cdot p^5 + 6 \cdot (1 - p^5)}{5} = \frac{6}{5} - p^5$$

In general, the formula becomes (for  $n$  components in series)

$$\frac{1 \cdot P(\text{ all pass } ) + (1 + n) \cdot P(\text{ at least one fails } )}{n} = \frac{1 \cdot p^n + (n + 1) \cdot (1 - p^5)}{5} = \frac{n + 1}{n} - p^n$$

or  $1 + \frac{1}{n} - p^n$  tests per component. (The book looks at this as *saving*  $p^n - \frac{1}{n}$  tests per component, which is equivalent.)

Let’s make a table for  $p = 0.95$  of the Number of Tests per Component. This is shown in Table 1. From this table we can see that testing  $n = 5$  components at a time will minimize the number of tests per component.

3 We can think of this as a Bernoulli trial if we assume that the transmission of each bit is an independent trial with probability of success  $p$ . The 7-bit code is successfully transmitted if at least 6 bits make it through without error. This has a probability of

$$b(7, 7; p) + b(7, 6; p) = \binom{7}{7}p^7 + \binom{7}{6}p^6q^1 = (0.99)^7 + 7(0.99)^6(0.01) = 0.99796895836506$$

or 99.796895836506%. (Compare this to the probability that 4 bits get through successfully:  $b(4, 4; .99) = 96.059601\%$ .)

Number of Components per Test	Number of Tests Per Component $p = 0.95$
$n$	$1 + \frac{1}{n} - p^n$
2	0.59750
3	0.47596
4	0.43549
5	0.42622
6	0.43157
7	0.44452
8	0.46158

Table 1: Testing Components For Problem 2