For each of the following differential equations...

(a) Write the equation in standard form $y' + P(x)y = G(x)$.

(b) Calculate an integrating factor $h(x) = e^{\int P(x)\,dx}$.

(c) Multiply the equation in part (a) by $h(x)$, and re-write this equation as $(\ h(x)y\ )' = h(x)G(x)$.

(d) Solve the differential equation by integrating both sides and solving for y.

(e) Find the solution with the given initial condition (for example, $y(1) = 5$ is the solution for which $y = 5$ when $x = 1$).

1 $y' + y = 3, \qquad y(0) = 1$

2 $y' + 2y = e^{-x}, \qquad y(0) = 5$

3 $xy' + 3y = 4, \qquad y(5) = 1$

4 $xy' - y = x^2e^x, \qquad y(1) = 0$

5 $y' + \dfrac{2}{x}\,y = 1, \qquad y(1) = 1$

6 $xy' + 3y = \dfrac{2}{x}e^{x^2}, \qquad y(1) = 0$

7 $2xy' + y = x, \qquad y(1) = 1$

8 $2xy' - y = 5x^3, \qquad y(1) = 3$

Answers:

1 $y = 3 - 2e^{-x}$ 2 $y = e^{-x} + 4e^{-2x}$ 3 $y = \frac{4}{3} - \frac{12^5}{3x^3}$ 4 $y = x(e^x - e)$

5 $y = \frac{x}{3} + \frac{2}{3x^2}$ 6 $y = \frac{e^{x^2}-e}{x^3}$ 7 $y = \frac{x}{3} + \frac{2}{3\sqrt{x}}$ 8 $= x^3 + 2\sqrt{x}$